

Investigating the Challenges of Data, Pricing and Modelling to Enable Agent Based Simulation of the Credit Default Swap Market.

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I, Laleh Zangeneh confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Abstract

The Global Financial Crisis of 2007-2008 is considered by three top economists the worst financial crisis since the Great Depression of the 1930s [Pendery, 2009]. The crisis played a major role in the failure of key businesses, declines in consumer wealth, and significant downturn in economic activities leading to the 2008-2012 global recession and contributing to the European sovereign-debt crisis [Baily and Elliott, 2009] [Williams, 2012]. More importantly, the serious limitation of existing conventional tools and models as well as a vital need for developing complementary tools to improve the robustness of existing overall framework immediately became apparent.

This thesis details three proposed solutions drawn from three main subject areas: Statistic, Genetic Programming (GP), and Agent-Based Modeling (ABM) to help enable agent-based simulation of Credit Default Swap (CDS) market. This is accomplished by tackling three challenges of lack of sufficient data to support research, lack of efficient CDS pricing technique to be integrated into agent based model, and lack of practical CDS market experimental model, that are faced by designers of CDS investigation tools. In particular, a general data generative model is presented for simulating financial data, a novel price calculator is proposed for pricing CDS contracts, and a unique CDS agent-based model is designed to enable the investigation of market. The solutions presented can be seen as modular building blocks that can be applied to a variety of applications. Ultimately, a unified general framework is presented for integrating these three solutions. The motivation for the methods is to suggest viable tools that address these challenges and thus enable the future realistic simulation of the CDS market using the limited real data in hand.

A series of experiments were carried out, and a comparative evaluation and discussion is provided. In particular, we presented the advantages of realistic artificial data to enable open ended simulation and to design various scenarios, the effectiveness of Cartesian Genetic Programming (CGP) as a bio-inspired evolutionary method for a complex real-world financial problem, and capability of Agent Based (AB) models for investigating CDS market. These experiments demonstrate the efficiency and viability of the proposed approaches and highlight interesting directions of future research.

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Chapter 1

Introduction

In the last five years, the world economy has been faced with one of the biggest crises ever seen, throwing most countries into recession. The causes of the financial meltdown are numerous, but it is widely accepted that one significant factor was the “Credit Default Swap (CDS)” contract. More research is vitally needed to analyze and define its impact more precisely and help financial market transparency.

To understand the CDS market, consider a small CDS market which comprises two categories of companies *Financial Institutions* and *Risky Companies*. The first category includes the wealthy financial institutions such as banks, hedge funds and insurance companies. The second category includes individual business companies such as airlines, car factories and IT companies. The financial institutions lend money to other companies to make money through interest. The risky companies are defined as companies who are seeking loans to survive and maintain their businesses.

This point is best illustrated with an imaginary example. North Bank (a financial institution) made a five-year \$10 million loan to West Airways (a risky company). North Bank is concerned about West Airways performance and not being able to pay back the loan (possible default). Therefore, in order to protect itself and reduce the risk of not getting its loaned money back, North Bank can buy a kind of insurance (known as “protection”) on West Airways from an insurance seller (a protection seller), which in this case might be East Bank (another financial institution). The insurance is based on a West Airways-issued bond (a debt security which represents a formal contract to repay borrowed money with interest at fixed intervals). This protection (insurance) contract is called a CDS contract and East Bank is then able to trade its CDS contracts with other banks, buying them when they cost less and selling them when they are worth more, in order to make profit. The price of the CDS contract changes according to the success or failure of the business of West Airways (i.e., the credit quality of West Airways). If the West Airways credit quality decreases (risk of default increases), the CDS price will increase.

In the language of Economics, a CDS contract is a kind of Credit Derivative (CD). A *CD* is a derivative security that has a payoff which is conditional on the occurrence of a credit event, explicitly designed to shift credit risk between the parties and its value is derived from the credit performance of one or more cooperation, sovereign or security [Francis et al., 2003]. Similarly, a CDS allows one to take or reduce credit exposure, commonly on bonds or loans of corporate entity and it reflects the risk of a default in a corporation. This risk is expressed through the CDS price [Beinstein et al., 2006]. Since the

CDS contract entered the financial market, trading of this complex financial product was unpredictable, out of control, and badly priced, leading to fortunes being made and lost [Navneet et al., 2005].

Although the CDS market concept sounds simple and convincing at the first sight, it involves enormous challenges in the real-world and there is no experimental tool available to study these challenges. In other words, this form of trading introduces new products such as CDS contracts into the market without the parties involved being aware of the possible impact of these products on the market's behaviour and future trends. Ideally, one should be able to study and to answer some principal questions about a proposed product before letting this product enter the market. For instance, in the case of the CDS contract, it would be valuable to study the associated potential problems or concerns related to the CDS contract, to identify the source of these problems to ascertain whether these problems might arise from the CDS concept and theory, and to assess the short-term or long-term impact of these problems on the market before exposing the market to this product. By being well informed about these problems, causes, and future consequences, appropriate action could be taken, such as banning the trade in this product or arranging the necessary measures to avoid catastrophic events.

However, in reality, the fundamental impacts of new products remain concealed until it is too late to take any action. This uncertainty is mainly because, on the one hand, the complexity of the world economy means the conventional methods and tools are largely ineffective or unreliable and, on the other hand, there is no other feasible tool available. Therefore, not surprisingly, we witness market events which do not match our beliefs and predictions, and by the time that these events occur we do not have any effective solution, nor any good understanding of the situation.

The 2007-2010 credit crisis set a new agenda for economists by highlighting the inefficiency of current macroeconomic models for spotting the problematic elements of the complex economic systems. Mr Jean-Claude Trichet, the president of European Central Bank (ECB), opened the 2010 ECBs flagship annual Central Banking Conference with a challenge to the scientific community to develop new approaches to understanding the economy [Trichet, 2010].

“When the crisis came, the serious limitations of existing economic and financial models immediately became apparent. Arbitrage broke down in many market segments, as markets froze and market participants were gripped by panic. Macro models failed to predict the crisis and seemed incapable of explaining what was happening to the economy in a convincing manner. As a policy-maker during the crisis, I found the available models of limited help. In fact, I would go further: in the face of the crisis, we felt abandoned by conventional tools.”

In July 2010, the wisdom of using a single, specific model for national economic policy was questioned in the US House of Representatives Committee of Science and Technology [Anon, 2010a]. The Institute for New Economic Thinking in New York, attacked many of the assumptions on which these models are based, including efficient financial markets and rational expectations [Anon, 2010a]. The two types of models generally considered to be most effective are Dynamic Stochastic General Equilibrium (DSGE) and Empirical Statistical Models [Farmer and Foley, 2009]. DSGE models assume a perfect

world but the 2007-2010 financial crisis illustrated how imperfect the world really can be. Empirical statistical models are capable of forecasting a few quarters ahead using historical data and assume that things stay more or less the same in the future - clearly not the case when the world goes through a turbulent time (2007-2010). Moreover, complicated mathematical tools used by Wall Street are designed to model the potential profit and risk of individual trades [Farmer and Foley, 2009] not the profit and risk of the global market. Mr Trichet also highlighted the danger of relying on a single tool in the annual ECB conference in November 2010 [Trichet, 2010].

“The key lesson I would draw from our experience is the danger of relying on a single tool, methodology or paradigm. Policy-makers need to have input from various theoretical perspectives and from a range of empirical approaches. ... We do not need to throw out our DSGE and asset-pricing models: rather we need to develop complementary tools to improve the robustness of our overall framework. ... In this context, I would very much welcome inspiration from other disciplines: physics, engineering, psychology, biology. Bringing experts from these fields together with economists and central bankers is potentially very creative and valuable. Scientists have developed sophisticated tools for analyzing complex dynamic systems in a rigorous way. These models have proved helpful in understanding many important but complex phenomena: epidemics, weather patterns, crowd psychology, magnetic fields. Such tools have been applied by market practitioners to portfolio management decisions, on occasion with some success.”

The complexity of the economic system does not only depend on financial products such as subprime mortgages and CDSs. These bring too much nonlinearity and complexity to handle and make things unpredictable [Farmer and Foley, 2009]. The actual factors for the financial crisis are considerably more complicated and can be traced back to human psychology [Keefe, 2008]. Greed is commonly a strong candidate for a cause [Anon, 2011] but the natural desire to seek risk [Keefe, 2008], a need for a sense of security (e.g. owning a home), a tendency to conform and follow leadership [Kaletsky, 2008], trust in the good will of others, and a heightened sense of self-interest and profit seeking [Keefe, 2008] are all contributing factors to the instability and unpredictability of the economic system. Therefore, as a result of limitations of the traditional macroeconomic model, the complexity of human psychology and Wall Street self-interest techniques, it seems that most economic policy-makers are using common sense and unreliable analogies to guide their decision-making and efforts to prevent crises [Farmer and Foley, 2009].

It seems clear that the assumption made by current macroeconomic models that things may stay more or less the same is unlikely to remain valid indefinitely, due to the complexity of the economic systems and human psychology. If such an assumption is breached, then the utility of current macroeconomics models is thrown into doubt.

What should replace the old methods is a problem still to be addressed. Agent Based (AB) models may provide a new approach. The potential of AB models for this task is increasingly becoming recognized. For example a workshop meeting, in Jun 2010, funded by Americas National Science Foun-

dation and attended by a diverse body of delegates that included economists from the Fed and the Bank of England, policy advisers and computer scientists, suggested AB models (ABMs) as an alternative tool for policy makers [Anon, 2010a]. The reason that ABMs became a valid candidate to this problem is that ABMs do not rely on the assumption that the environment will move towards a predetermined equilibrium state, therefore, it overcomes some of the limitations of the other models (e.g. DSGE) [Farmer and Foley, 2009]. In addition, AB modelling has successfully helped policy-making in epidemiology and traffic control [Farmer and Foley, 2009]. This approach reveals a novel and challenging research problem, which will be discussed in more detail next.

1.1 Research Problem

In this thesis, the problems of existing CDS market experimental tools are tackled by investigating viable approaches for developing an experimental CDS market in which the real-world data and CDS pricing techniques are taken into account. A sufficient tool to study and to investigate the individual traders and the emergent market behaviour as well as studying the impact of different strategies and controls on the whole system would help to diagnose the critical and risky products at the early stages and avoid future crises. It is to be expected that such a tool would help to answer some fundamental questions, or at least shed substantial new light on them. Moreover, unlike current forecasting models, this experimental tool could warn in the case of an unusual situation to avoid manageable losses and future financial crisis.

In this work, three main challenges involved in the creation of the tool are identified.

Challenge 1: Lack of sufficient data to support research. Most attempts to apply AB models to financial market issues have been conceptual to date, and have not been parameterized, calibrated with, or tested on real data. Moreover, since CDS entered the financial market, the lack of sufficient data had been a major problem for a broad empirical testing of CDS models such as CDS pricing models [Hull and White, 2000, Houweling and Vorst, 2005] until a few years ago. In the last few years with increased bond market liquidity and a well-developed CDS, the market has provided more than sufficient data for investigation. However, the period of this historical data is still not sufficient for building a long simulation (e.g. 20+ years).

Challenge 2: Lack of efficient CDS pricing technique to be integrated into agent based model.

In the CDS literature, the Duffie approach [Hull and White, 2000] is referred to as a theoretical method to evaluate the correct pricing of the CDS's spread through a simple relationship. Nevertheless, observing the CDS spread, debt return and risk free rate in the financial market shows that this relation does not hold exactly due to arbitrage¹ cost [Francis et al., 2003]. Pricing of a CDS contract is a challenging open problem that is both a quantitative and qualitative field involving estimations of default, timing of default and balance sheet value fluctuations [Joro et al., 2004, Anon, 2008].

Challenge 3: Lack of practical CDS market experimental model. As discussed earlier, the current traditional microeconomic models fail to comprehensively or successfully study the market com-

¹ Arbitrage is the practice of taking advantage of a price difference between markets.

ponents and behaviour. Consequently, the market becomes exposed to unpredictable tragedies. A final and preeminent challenge is to provide an experimental tool for the CDS market. The principle issue is how best to model the CDS market, to deliver a feasible experimental tool in which the different market participants, elements, behaviours and events can be investigated under various laboratory conditions and situations, thus increasing overall market understanding and transparency.

1.2 Research Aim

The CDS market is unpredictable, volatile, and prone to wild variations in profit and loss. There are no practical computational tools in existence that can be used for studying market behaviour and making useful predictions. Therefore, our main objective is to investigate viable approaches that address the three challenges associated with tackling this problem (lack of sufficient data to support research, lack of efficient CDS pricing technique to be integrated into agent based model, and lack of practical CDS market experimental model) and demonstrating the utility of such a combination of approaches for investigating significant questions about the CDS market. Answering these questions will serve as a proof of principle that the approaches identified in this work are useful. The overall aim of the thesis is:

“To investigate the viability of approaches that address the three challenges of lack of sufficient data to support research, lack of efficient CDS pricing technique to be integrated into agent based model, and lack of practical CDS market experimental model, that are faced by designers of CDS investigation tools.”

where:

Viability will be measured through an assessment of the practicality and feasibility of approaches for the three key challenges. Evidence will be provided through analysis of the literature and empirical studies.

Approaches. Three classes of approach will be considered, one for each challenge. Approaches will include (1) methods for cleaning and sampling real world data, stochastic processes for simulating real world data; (2) different regression techniques for CDS pricing; (3) methods such as AB modelling for simulating the CDS market.

1.3 Scope and Objectives

Studying and investigating the entire world economy is a gigantic task which is beyond the scope and scale of a single PhD. Therefore, this work focuses on one of the most influential (and arguably, damaging) markets in the world economy, the CDS market. The focus throughout this work is on the CDS market, specifically the CDS energy market as the availability of financial data for this sector makes this feasible. This financial data includes the CDS and bond price information as well as the interest rate over the period of five years.

Thus, throughout this thesis, when it refers to “CDS market” it specifically refers to the CDS energy market, and specifically the part of the market which focuses on CDS deals, therefore we are not modelling the process of loan deals. The key participants of CDS market are considered: CDS dealer (buyers or sellers), CDS reference entities (risky companies), credit rating agencies, and financial service authority (government). However, restrictions are applied in the design process to market participants’ concepts, behaviours, and actions. For instance, the traders’ business deals are limited to lending money, buying CDS contracts, and selling CDS contracts and the process of lending money to risky companies is simplified. This restriction is mainly for the purpose of diminishing the unnecessary complexity of the model, so that we can observe and inspect the important activities and their impacts on environment more readily.

This work will not consider evidence gathered through comparative studies for there may be no clear sense in which one approach is better or worse than another because different modelling approaches may solve the challenges with very different and incompatible methods.

Consequently, the aim of this thesis is broken down into three objectives:

- Studying the challenge of simulating data with a focus on incorporating the real-world data characteristics, integrating a market common trend, and overcoming the problem of limited data length.
- Studying the challenge of the CDS pricing with a focus on providing a price calculator tool which can be integrated with a multi-agent system.
- Studying the challenge of modelling the CDS market using AB simulation while utilizing complementary tools, and with a focus on simulating market participants, events, and participants’ negotiation processes.

1.4 Contributions

This thesis contributes to the field of Agent-Based Computational Economics (ACE) by investigating the viable approaches for developing an AB CDS market in which the real-world data and CDS pricing technique are taken into account. This work is the first published step toward studying the CDS market using AB modelling and evolutionary techniques. Within this research stream, and in the course of meeting the stated objectives, this work makes the following main contributions:

- A stochastic process is developed for designing a data generative model that uses the attributes of real-world data in order to generate unlimited financial data with the purpose of overcoming the lack of real-world data for long duration simulations (20+ years), as well as generating different scenarios and an unlimited number of samples. This is the first time that this technique has been used for feeding an AB model.
- A common market factor is introduced to the developed data generative model in order to simulate an industry sector behaviour.
- A CDS price calculator is designed to be used by modelled traders in the virtual economy in which the best function for pricing each individual CDS contract is discovered based on the available

financial history (e.g. bond price and interest rate) of a referenced company. This is a novel demonstration of cooperation between ABM and the evolutionary regression process.

- A study comparison of an evolutionary and a non-evolutionary regression tool is provided for pricing CDS contracts and their compatibility with AB models.
- A CDS market experimental model is designed to enable CDS market investigation. This study is the first time that an AB model has been utilized for studying the CDS market fundamentals, behaviour, and events.
- A novel methodology for incorporating empirical data into ABM is proposed.
- A chromosome reader function is derived from the original Cartesian Genetic Programming (CGP) program. This function has been added to the Credit Default Swap Agent-Based model and acts as a translator to enable CDS contracts price calculation by the modelled traders.
- A unified general framework is presented for integrating these three solutions in order to enhance the capabilities, efficiency and practicability of this Credit Default Swap Agent-Based model.

1.5 Publications

The following publications have resulted from this research, and will be referenced where appropriate in the text.

Zangeneh, L. and Bentley, P.J. (2009) Cartesian Genetic Programming to find a best regression model between CDS spread and Bond Yields. Abstract in International Conference on Computational and Financial Econometrics. October 2009. [Zangeneh and Bentley, 2009]

Zangeneh, L. and Bentley, P. J. (2010) Analyzing the CDS Market Using Cartesian Genetic Programming. Proc of 11th International Conference on Parallel Problem Solving From Nature. September 11-15, 2010. AGH University of Science and Technology, Krakow, Poland. Springer LNCS. [Zangeneh and Bentley, 2010]

Zangeneh, L. and Bentley, P.J. (2014) To Evolve or Not to Evolve: An Investigation of CGP and GPR for CDS Pricing. In preparation for journal of Evolutionary Computing. [Zangeneh and Bentley, 2014b]

Zangeneh, L. and Bentley, P.J. (2014) CDS Agent-based Virtual Economy (CAVE). In preparation for journal of Autonomous Agents and Multi-Agent Systems. [Zangeneh and Bentley, 2014a]

1.6 Organization

The rest of this thesis is structured as follows:

Chapter 2 critically reviews the related literature from several distinct areas of research, including AB modelling, regression, stochastic process, and economic that underlie the theoretical and algorithmic developments which outline this thesis.

Chapter 3 presents the model framework, as well as detailing the research problem that this thesis focuses on.

Chapter 4 presents a novel solution for providing a generative data engine which studies the empirical data, extracts its attributes and produces data with similar attributes for an selected period of time.

Chapter 5 presents a proposed bio-inspired evolutionary CDS price calculator, which takes the bond price and interest rate as inputs and provides the price of a CDS contract.

Chapter 6 introduces an AB approach for simulating a CDS market with a focus on simulating market participants, market events and the negotiation process of the participants.

Chapter 7 presents a unified general framework for integrating the three approaches (Chapters 4, 5, and 6) to form an AB model which is closer to reality and thus a more practical tool for dealing with real world challenges.

Chapter 8 contains the concluding remarks and proposes directions for future work in the field of ACE.

Chapter 2

Background

In recent years, the research techniques for economists have been widely developed as a result of substantial advances in modelling tools [Tsfatsion, 2003, Arthur et al., 1997, Batten, 2000]. Therefore, the quantitative modelling of a wide variety of complex phenomena associated with decentralized market economies has become popular among researchers, e.g., inductive learning, imperfect competition, endogenous trade network formation, and the open-ended co-evolution of individual behaviours and economic institutions [Tsfatsion, 2002]. Agent-Based Computational Economics (ACE) is a branch of this field. This thesis falls into the field of ACE. This chapter will provide background information for the four main subject areas that form the basis of this thesis: Credit Default Swap (CDS), ACE, Data analysis, and CDS Pricing Techniques. CDS is a kind of unfunded Credit Derivative (CD) [O’Kane, 2001] which became recognised as one of the main causes of the 2007-2010 financial crisis. ACE is the computational study of economies modelled as evolving systems of autonomous interacting agents under controlled experimental conditions [Tsfatsion, 2003]. Data analysis is a process of studying, cleaning, transforming and modelling data with the aim of highlighting useful information, describing facts, detecting patterns, suggesting conclusions and supporting decision making. CDS Pricing Techniques is the body of methods that help to value the CDS contracts. Where these research areas intersect (see the golden point in Figure 2.1), interesting challenges can be found that drive development in the ACE field.

The rest of this chapter is structured as follows: in Section 2.1, there is a comprehensive background on CDS. Particularly, the focus is on the primary attributes of the CDS contract, CDS risk and trading, market participants, and market structure. A brief review of Agent Based (AB) modelling is provided in Section 2.2, where the focus is on AB modelling methodology, theory, and application. Section 2.3 is focused on AB data analysis related literature which is specifically introduced for incorporating empirical data into the AB model. Before reviewing these solutions, how AB models are traditionally catered for by data is examined. The CDS pricing techniques are explained in Section 2.4 where the traditional architectures are described outlining each of these and their suitability and utility for AB modelling. In Section 2.5, the focus is on the state-of-the-art of ACE, where the research undergone so far in the field is reviewed, describing its practicability for the financial market, and detailing some of the major relevant works in this field. Section 2.6 summarises the background discussion and highlights the critical issues of each subjects.

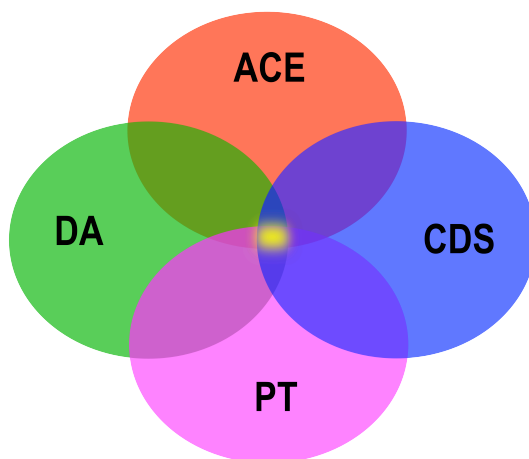


Figure 2.1: Intersection of four main research areas that forms the basis of thesis thesis, where *CDS* refers to Credit Default Swap, *ACE* stands for Agent-based Computational Economics, *DA* refers to Data Analysis, and *PT* stands for Pricing Techniques.

2.1 Credit Default Swap

A Credit Default Swap (CDS) is a type of unfunded CD. A CD is a derivative security that has a pay-off which is conditioned on the occurrence of a credit event, explicitly designed to shift credit risk between the parties with a value derived from the credit performance of one or more cooperation, sovereign or security [Francis et al., 2003]. CDS contracts have existed in the market since the early 1990s [Smithson et al., 2006], and increased in use after 2003 [Anon, 2010b]. Most CDSs are in the \$10-\$100 million range with maturities between one and 10 years [Schönbucher, 2003]. CDS can be used by investors for speculation¹, hedging and arbitrage purposes [Nakisa, 2011]. The CDS is uniquely defined by four key parameters [Brothers, 2003, Francis et al., 2003, Beinstein et al., 2006]:

- **Issuer:** CDS contracts specify a reference bond or loan which defines the issuing entity through the bond prospectus (e.g. West Airways in the example given at the start of the first chapter).
- **Notional amount:** Notional amount is the amount of credit risk being transferred between protection buyer (e.g. North Bank) and protection seller (e.g. East Bank).
- **Spread:** A spread (also called coupon, or price) specifies the annual payments which are quoted in Basis Point (bp)². These payments are paid quarterly (e.g. from North Bank to East Bank).
- **Maturity Date:** The expiration of the contract. The most liquid³ maturity term for CDS contract is 5 years.

A CDS is defined as a swap contract and agreement in which the protection buyer of the CDS (seller of credit risk) makes a series of payments (often referred to as the *CDS fee* or *spread*) to the protection

¹The act of trading in an asset, or conducting a financial transaction, that has a significant risk of losing most or all of the initial outlay, in expectation of a substantial gain. See: <http://www.investopedia.com>

²A basis point is a unit relating to interest rates that is equal to 1/100th of a percentage point per annum (pa).

³Liquid means easily converted into cash (e.g. a bond which can be sold quickly).

seller (buyer of credit risk) who, in exchange, receives a payoff if a credit instrument (typically a bond or loan) experiences a credit event. In its simplest form, a CDS is a bilateral contract between the buyer and seller of protection. The reference entity (also called reference obligor), which a CDS refers to, is usually a corporation or government and it is not a party to the contract. The protection buyer makes quarterly premium payments, the “spread”, to the protection seller.

The premium payments are generally quarterly, with maturity dates falling on 20th March, 20th June, 20th September, and 20th December. Due to the proximity to the IMM dates, which fall on the third Wednesday of these months, these CDS maturity dates are also referred to as “IMM dates” [Nakisa, 2011]. If the reference entity defaults, the protection seller pays the buyer the par value of the bond in exchange for physical delivery of the bond, although settlement may also be by cash or auction [Weistroffer, 2009]. A default is referred to as a “credit event” and includes such events as failure to pay, restructuring and bankruptcy [Weistroffer, 2009].

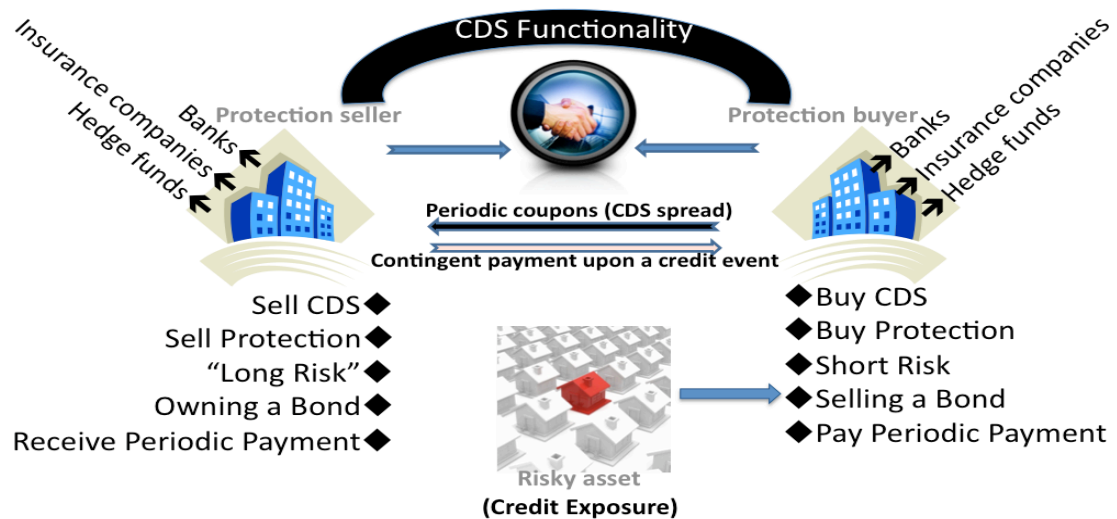


Figure 2.2: Single Name CDS Functionality

Figure 2.2 illustrates the terminology and mechanism of the CDS. To better explain this illustration the fictional example mentioned earlier will be recalled. In this case, North Bank made a five-year \$10 million loan to West Airways. North Bank is concerned about West Airways performance and not being able to pay back the loan (possible default). Therefore, in order to protect itself and reduce the risk of not getting its loaned money back, North Bank can buy a kind of insurance (known as “protection”) on West Airways from a insurance seller (a protection seller), which in this case might be East Bank. The insurance is based on a West Airways-issued bond (a debt security which represents a formal contract to repay borrowed money with interest at fixed intervals). This protection (insurance) contract is called a CDS contract and East Bank is then able to trade its CDS contracts with other banks, buying them when they cost less and selling them when they are worth more, in order to make profit. The price of the CDS contract changes according to the success or failure of the business of West Airways (i.e. the credit quality of West Airways). If the West Airways credit quality decreases (risk of default increases), the CDS price will increase.

As mentioned above, CDS is an agreement between two parties to exchange the credit risk of a reference entity, also called an issuer (West Airways, in this example), however it does not directly involve the issuer [Francis et al., 2003]. The protection seller profits if the credit of the reference entity remains stable or improves while the swap is outstanding [Beinstein et al., 2006].

2.1.1 Is CDS an Insurance Contract?

CDS contracts have been compared with insurance, because the buyer pays a premium and, in return, receives a sum of money if a default occurs. However, there are a number of differences between CDS and insurance. Unlike insurance, the CDS seller does not have to be a regulated entity and it is not required to maintain any capital reserves to guarantee payment of claims. The CDS buyer does not need to own the underlying security. CDS dealers manage risk by hedging CDS and insurers manage risk mainly by setting loss reserves based on the Law of Large Numbers (LLN). The LLN is a theorem (in probability theory) that explains the outcome of performing one experiment a large number of times⁴. An insurance contract provides an indemnity against the losses actually suffered by the policy holder, whereas the CDS provides an equal payout to all holders.

There are also important differences in pricing approaches. For instance, to cancel the insurance contract the buyer can simply stop paying premiums whereas in the case of CDS the protection buyer may need to unwind (exit) the contract by negotiation which might result in a profit or loss situation. Finally, the insurance contracts require the disclosure of all known risks involved, but CDSs have no such requirement [Garbowski, 2008, Cox, 2008, Frielink, 2008].

2.1.2 Terms of a Typical CDS Contract

A CDS contract is typically documented under a confirmation referencing the CDs definitions as published by the International Swaps and Derivatives Association. The confirmation typically states [Anon, 2003a]:

- **A reference entity** (also called a corporation or sovereign) that generally, although not always, has debt outstanding.
- **A reference obligation** usually an unsubordinated corporate bond or government bond.
- **Default protection period** which is defined by the contract effective date and scheduled termination date.
- **A calculation agent** who is responsible for making determinations as to successors and substitute reference obligations and for performing various calculations and administrative functions in connection with the transaction⁵ [Nakisa, 2011].

⁴See [Verhoeff, 1993] for more information about Law of Large Numbers.

⁵“By market convention, in contracts between CDS dealers and end-users, the dealer is generally the calculation agent, and in contracts between CDS dealers, the protection seller is generally the calculation agent. It is not the responsibility of the calculation agent to determine whether or not a credit event has occurred but rather a matter of fact that, pursuant to the terms of typical contracts, must be supported by publicly available information delivered along with a credit event notice. Typical CDS contracts do not provide an internal mechanism for challenging the occurrence or non-occurrence of a credit event and rather leave the matter to the courts if necessary, though actual instances of specific events being disputed are relatively rare.” [Nakisa, 2011]

- **Credit events** that terminate the contract in which the payment obligations by the protection seller and delivery obligations by the protection buyer should be made.
- **Deliverable obligation characteristics** that specify the range of obligations that a protection buyer may deliver if a credit event occurs.

2.1.3 CDS Risk and Trading

By entering into a CDS contract, both the protection buyer and protection seller of credit take on counterparty risk [Mengle, 2007, Weistroffer, 2009]. The protection buyer takes the risk that the seller will default and the protection seller takes the risk that the buyer will default on the contract, depriving the protection seller of the expected revenue stream. Another kind of risk for the protection seller of CDS is jump risk or jump-to-default risk. This risk is not present in other over-the-counter derivatives [Weistroffer, 2009]. In addition, CDS might involve liquidity risk.

The value of each CDS contract fluctuates based on increasing or decreasing probability that the underlying asset will have a credit event. When the default probability of the underlying asset increases, the contract is worth more for the protection buyer and worth less for the seller. The opposite occurs if the probability of default decreases. Therefore, these contracts are regularly traded instead of being kept until the expiration date. For instance, a trader in the market might speculate that the credit quality of a reference entity will deteriorate at some time in the future and will buy protection for a very short term in the hope of profiting from the transaction. An investor can exit from a contract by selling his or her interest to another party, offsetting the contract by entering into another contract on the other side with another party, or offsetting the terms with the original counterparty. Because CDSs are traded Over-the-counter (OTC)⁶, involve intricate knowledge of the market and the underlying assets and are valued using industry computer programs [Pinsent, 2012]; they are better suited for institutions⁷ (e.g. Association of British Insurers) rather than retail investors⁸.

2.1.4 Modeling Corporate Default Risk

In the credit risk literature, there are two broad complementary approaches to modelling corporate default risk (e.g. the risk that West Airways defaults on a loan): the *Structural Approach* and the *Reduce-Form Model*. In the structural model the evolution of the company's assets follows the diffusion process. In other words the default occurs when the value of the firm assets becomes lower than its debts; because the assets can be continuously assessed, downwards trends can be spotted and so the risk of default should never be a surprise. In contrast to the structural approach, the reduce-form approach assumes that there is no relation between the value of the company and the risk of default. In this approach defaults are seen as unpredicted Poisson events involving a sudden loss in market

⁶The phrase "over-the-counter" (OTC) is used to refer to stocks, debt securities and other financial instruments such as derivatives, which are traded through a dealer or directly between two parties as opposed to on a centralised exchange (e.g. London Stock Exchange).

⁷An institutional investor is an investor (e.g. insurance company, hedge fund, or mutual fund) that is financially sophisticated and makes large investments.

⁸Individual investors who buy and sell securities for their personal account, and not for another company or organization.

value and, therefore, firms never default gradually. Below we summarise these two approaches. See [Navneet et al., 2005, Black and Cox, 1976, Hofberger and Wagner, 2008, Elizalde, 2003] for more details on credit risk literature. Table 2.1 summarizes the history of credit risk approaches by referring to the investigators who have contributed to the field.

Table 2.1: History Track of Credit Risk Approaches

Structural Model		Reduced-form Model	
Investigators	Date	Investigators	Date
Black & Scholes	1973	Geske, Ingersoll, Merton	1977
Merton	1974	Smith & Warner	1979
Black & Cox	1976	Cooper & Mello	1991
Longstaff & Schwarts	1995	Hull & White	1992
		Abken	1993
		Duffie & Singleton	1995

Structural Approach

In 1974, Merton proposed that the evolution of the firm's assets follows the diffusion process. In other words, the defaults accrue when the value of the asset becomes lower than the debt and firm never defaults by surprise due to the diffusion process which is continuous; thus, a sudden drop in firm's value is impossible. Merton modelled a firm's asset value as a lognormal process and assumed that the firm would default if the asset value, A , falls below a certain default boundary X . The default was allowed at only one point in time, T . The equity, E , of the firm was modelled as a call option on the underlying asset. The value of the asset was given as: [Navneet et al., 2005]

$$E = A\Phi[d_1] - X_T \exp[-rT]\Phi[d_2] \quad (2.1)$$

Where

$$d_1 = \frac{\log[\frac{A}{X}] + (\mu + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T} \quad (2.2)$$

and Φ represents the cumulative normal distribution. The debt value, D , is then given by $D = A - E$. The spread can be computed as:

$$S = -\frac{1}{T} \log \Phi[d_2] + \frac{A}{X} \exp[rT]\Phi[-d_1] \quad (2.3)$$

Where A is the initial asset value of the firm, X is the default barrier for the firm, μ is the drift of the asset return, and σ is the volatility of the asset returns.

Reduced-Form Approach

In contrast to the structural approach, the Reduced-form approach assumes that there is no relation between the firm's value and default. In this approach a default is seen as an unpredicted Poisson event involving a sudden loss in market value and, therefore, the firm never defaults gradually. This approach

was proposed by Duffie in 1998 and the idea behind this approach is that the credit event can be (approximately) modelled as a Poisson process, with a hazard rate (or intensity rate) h depending on the length of the time interval. This implies that the probability of observing a credit event between time 0 and time t is equal to $h * t$. The Duffie approach is defined as follows (See [Esposito, 2002] for the full details of this approach):

The general form of Poisson distribution is, for $x = 0, 1, 2, \dots$:

$$\text{Poisson}(x; h) = \frac{e^{-ht} (ht)^x}{x!} \quad (2.4)$$

Where x denotes the number of credit events.

For a single firm the significant situation is a case where $x = 0$ which means the firm does not default whereas for $x > 0$, the firm defaults. The probability that the firm defaults before time t (the so called waiting time or hitting time or if reversed survival time) is exponentially distributed:

$$\text{PD}(t) = 1 - e^{-ht} \quad (2.5)$$

Differentiating $\text{PD}(t)$ with respect to t , gives the density function expression:

$$\text{pd}(t) = he^{-ht} \quad (2.6)$$

Parameter h measures the expected life. For instance, where $h = 0.1$, we expect that the credit event will happen in 10 years' time. Another useful property of a Poisson process with hazard rate h , is that the time until it is observed that x is happening is distributed as a gamma function with parameter (h, x) . The function form of the gamma density function is:

$$\text{pd}(t, T, x) = \frac{h^x}{\gamma(x)} (T - t)^{x-1} e^{-h(T-t)} \quad (2.7)$$

If $x=1$, the exponential case is relevant.

Structural versus Reduced Form Approach

A study by Jarrow and Protter [Jarrow and Protter, 2004], in 2004, claims that the fundamental difference between the structural and the reduced-form approach comes from the information set available to the modeller. Structural models rely on very detailed information typically held by the firm's insiders and reduced form models rely on less detailed information typically observed by the market. Thus, both models should show a similar performance in the case of similar market information. An empirical investigation by Yalin Gaundauz and Marliese Uhrig-Homburg [Gunduz and Uhrig-Homburg, 2011] confirms that they have the same result. The authors claim that, in the case of CDS pricing, both models mostly underpredict spreads and underprediction typically increases as the credit-rating worsens and maturity increases. Moreover, the reduced-form approach outperforms the structural for investment-grade names and longer maturities. In contrast, the structural approach performs better for shorter maturities and sub-investment grade names.

2.1.5 Market participants

The CD market consists of an end-buyer of protection, an end-seller of protection and intermediates. End-buyers of protection are entities that seek to hedge credit risk taken in other parts of their business. The predominant entity in this group is commercial banks. There are also insurance funds, pension funds, and mutual funds that seek protection for credits held in their portfolio. End-sellers of protection are entities that seek to diversify their current portfolio and can do so more efficiently with CDs. An entity that provides protection is seeking exposure to a specific credit or a basket of credits. Intermediates include investment banking arms of commercial banks and securities houses. Their key role in the CD market is to provide liquidity to end-users. They trade for their own account looking for arbitrage and other opportunities [Brothers, 2003, Francis et al., 2003].

The majority of CDs take the form of CDS which transfer the default risk of one or more reference entities from one party to another. CDS make-up 60% of CDs market when credit index products make-up 25% and other derivatives make-up 15% of the market [Anon, 2005].

2.1.6 CDS Market Structure

The CDS market consists of two sub-markets. The first sub CDS market includes parties where the buyer of the CDS (protection buyer) also holds the underlying credit asset (loan or bond). The second sub market developed when speculation entered the market and new parties became involved in the CDS market without owning the underlying asset. These new parties gamble on the possibility of a credit event of a specific asset. Speculation turns out to be the major problem of the CS market. Many banks, hedge funds and investment institutions started to write CDS contracts and gamble on whether a credit event would occur. These CDS contracts created a way to make money from a downward bonds market⁹ [Sheffrin, 2003]. Many hedge funds and other investment companies often gamble on the price movement of commodities, interest rates and many other items, and they now had a vehicle to sell to the credit markets [Zabel, 2008].

2.1.7 Sources of Market data

The CDS market data is available from three main sources. The International Swaps and Derivatives Association (ISDA) provides data on an annual and semi-annual basis since 2001.¹⁰ The Bank for International Settlements (BIS) provides data on the same basis since 2004.¹¹ The weekly data is provided by the Depository Trust & Clearing Corporation (DTCC), through its global repository Trade Information Warehouse (TIW), however the publicly available information goes back only one year.¹² The Office of the Comptroller of the Currency publishes quarterly CD data about insured U.S commercial banks and trust companies.¹³ Each provider uses a different sampling technique, therefore the numbers provided by each source do not always match. According to DTCC, the Trade Information Warehouse maintains the only global electronic database for virtually all CDS contracts outstanding in the marketplace.

⁹A downward market is a general decline in the stock market over a period of time (also called a “bear market”)

¹⁰“Surveys & Market Statistics”. International Swaps and Derivatives Association (ISDA). Retrieved April 20, 2010.

¹¹“Regular OTC Derivatives Market Statistics”. Bank for International Settlements. Retrieved April 20, 2010.

¹²“Trade Information Warehouse Reports”. Depository Trust & Clearing Corporation (DTCC). Retrieved April 20, 2010.

¹³“Publications: OCC’s Quarterly Report on Bank Derivatives Activities”. Office of the Comptroller of the Currency.

[Nakisa, 2011]

2.1.8 Discussion

The CDS has become well-known as one of the causes of the 2007-2010 crisis. An article by Matthew Philips in September 2008 explains how CDS becomes dangerous while it was supposed to be an insurance against risky loans [Philips, 2008]. Some of the main CDS problems highlighted by the Leaders of the Group of Twenty in November 2008 are [Anon, 2008]:

- OTC derivatives including CDS have high systemic risks.
- OTC derivatives including the CDS market lacks from an electronic trading or exchange traded platforms.
- OTC derivatives including the CDS market suffers from low transparency.
- OTC derivatives including the CDS market suffers from low infrastructure support for growing market volumes.

The risk of counterparties defaulting has been intensified during the recent financial crisis, particularly because Lehman Brothers and AIG were involved as counterparties in a very large number of CDS transactions. This is an empirical example of systemic risk, risk which threatens an entire market, and a number of commentators have claimed that size and deregulation of the CDS market have increased this systemic risk [Nakisa, 2011] and exacerbated the 2008 global financial crisis by hastening the demise of companies such as Lehman Brothers and AIG [Philips, 2008]. In the case of Lehman Brothers, it is claimed that the bank faced a series of problems which it was not able to overcome as a result of a lack of confidence due to widening of its CDS spread. However, proponents of the CDS market argue that the CDS spread simply reflected the reality that the company was in serious trouble and the investors who had counterparty risk with Lehman Brothers were allowed to reduce their exposure in the case of their default.

The CDS market has grown significantly over the last fifteen years. The global market for CDS increased from \$300 billion in March 1998 [Tett, 2009] to over \$2 trillion on 2002 [Anon, 2010b]. The CDS market size more than doubled in size each year from \$3.7 trillion in 2003. The notional value was stimulated \$62.2 trillion by the end of 2007 where the world stock markets capitalization¹⁴ was just about \$61 trillion¹⁵ at the same time. However, the notional amount outstanding had fallen 38 percent to \$38.6 trillion by the end of 2008 [Anon, 2010b], and fell to \$26.3 trillion by mid-year 2010¹⁶.

Critics claimed that the CDS market became too large without proper regulation [Philips, 2008] and infrastructure support [Anon, 2008]. Because of privately negotiated contracts, the market has no

¹⁴Market capitalization (or market cap) is the total value of the issued shares of a publicly traded company; it is equal to the share price times the number of shares outstanding.

¹⁵World Stock Market Value Reaches 20-Month High available at: <http://seekingalpha.com/article/199294-world-stock-market-value-reaches-20-month-high>

¹⁶Based on ISDA 2010 MID-YEAR MARKET SURVEY available at: <http://www.isda.org/statistics/recent.html>

transparency. Due to a lack of transparency, the protection buyers and protection sellers were unknown, and consequently they were left out of the negotiation process in a critical situation¹⁷.

2.2 Agent Based Modelling

Traditionally, econometric and Dynamic Stochastic General Equilibrium (DSGE) models are utilized by the world economy for the purpose of market investigation and forecasting [Farmer and Foley, 2009]. An econometric model is a statistical model which specifies the statistical relationship that is believed to hold between the various economic quantities [Sims, 1980]. DSGE modelling is a branch of applied general equilibrium theory which aims to explain the behaviour of the economy as a whole by analysing the interaction of many microeconomic decisions [Tovar, 2008]. Despite the value of the existing models, these models are limited for several reasons. Current econometric models are entirely dependent on historical data, thus cannot provide insight into circumstances that are novel. DSGE models employ extremely restrictive assumptions (e.g. a perfect world), are highly aggregated, and are linear for reasons of analytic tractability [Farmer and Foley, 2009]. These models have historically not considered the institutional and financial structures needed for the modern economy, thus they are not viable when the world goes through a turbulent time, and cannot provide insight into conditions that occurred in the recent crisis such as housing bubbles and credit crunches. As believed and pointed out by Mr. Trichet, the president of European Central Bank (ECB), business and credit cycles are indeed caused by endogenous nonlinear oscillations of the economy, consequently DSGE models will never provide what is needed to accurately model them [Trichet, 2010]. Significantly, it is not just Mr. Trichet who seeks a new solution [Trichet, 2010] - senior policymakers in finance and economic ministries, central banks, and regulatory agencies across the world [Anon, 2010a] are looking for the same solution and this search is a reflection of the gravity of the recent crisis and unpromising feasibility of the existing tools. The potential of AB models, for addressing the current traditional models limitations, is increasingly becoming recognised.

AB modelling (also called AB systems, multi-agent system or multi-agent simulation) is a class of computational models for simulating the actions and interactions of autonomous agents with an aim of assessing their effects on the system as a whole. AB modelling is a relatively new modelling paradigm and is one of the most exciting practical developments in modelling since the invention of relational databases [Tessfatsion, 2003]. AB modelling promises to have far-reaching effects on the way that businesses use computers to support decision-making and researchers use electronic laboratories to support their research. AB modelling is a multi-disciplinary field and has connections to many other fields including complexity science, systems science, Systems Dynamics, computer science, management science, the social sciences in general, and traditional modelling and simulation [Macal and North, 2005]. The key principle behind the AB modelling is that simple behavioural rules generate complex behaviour. This is a design principle articulated by Kelly Johnson, known as “Keep It Simple Stupid (K.I.S.S.)”, extensively adopted in the modelling community. There is no universal agreement on the precise definition of the term *agent*, but from a practical modelling point of view, an agent should be identifiable, situated,

¹⁷See the discussion by Masaccio on 14 May 2009: Protecting GM from Credit Default Swap Holders. Available at: <http://firedoglake.com/2009/05/14/protecting-gm-from-credit-default-swap-holders/>

goal-directed, autonomous, self-directed, flexible, and adaptable [Macal and North, 2005]. Some modellers consider any type of independent component (software, model, individual, etc.) to be an agent [Bonabeau, 2002], while others insist that a component's behaviour must be adaptive in order for it to be considered an agent [Mellouli et al., 2004]. Casti [Casti, 1997] argues that agents should contain both base-level rules for behaviour as well as a higher-level set of rules to change the rules". Jennings [Jennings, 2000] provides a computer science view of agency emphasizing that the fundamental characteristic of an agent is defined as its capability to make independent decisions. Generally speaking an agent can be defined as an independent component with an adaptive behaviour and decision making capability.

2.2.1 The classical ABM methodology

The classical AB modelling methodology has been founded on methodology described as a "*logic of simulation*" [Gilbert and Troitzsch, 2005]. The logic of simulation methodology consists of four key processes: *Abstraction*, *Simulations*, *Data gathering* and *Structural similarities* defined as below:

Abstraction is a process of driving higher concepts from an observed target while hiding away the details for the sake of reducing the complexity to obtain a more understandable model of the target.

Simulations is a process of running a model forward over time and observing what happens. The initial conditions (the state in which the model starts) are always important as, often, the dynamics of the model are strongly dependent on the precise initial conditions.

Data gathering is a process of collecting data (e.g. qualitative, quantitative, or both) from a real phenomenon. Depending on the phenomenon, data can be obtained from various sources such as surveys, measuring processes, official documents, internet records and data providers (e.g. Thomson Reuters¹⁸ or national Data Archives¹⁹)

Structural similarities, here, refers to the process of measuring the similarities between two data sets (e.g. between observed data and simulation result).

Figure 2.3 presents a diagrammatic definition of the logic of simulation. As illustrated, the model designed based on the abstraction of the observed target. The target is a dynamic entity, changing over time and reacting to its environment. The result obtained from simulations is called Simulation data. The collected data (simulation target) is obtained through the process of data gathering. Finally, the process of analyzing the structural similarity between the simulated data and collected data is used for validation purposes. The structural similarity expresses how well or how poorly the AB modelling can simulate the real observed phenomenon. The simulation data might be improved by modifying the model design and structure.

¹⁸Thomson Reuters Corporation is a business data provider. It operates in two divisions: Professional (Legal, IP & Science and Tax & Accounting) and Markets (Financial Professionals & Marketplaces, Enterprise Solutions and Media).

¹⁹European archives are listed at <http://www.nsd.uib.no/cessda/archives.html>

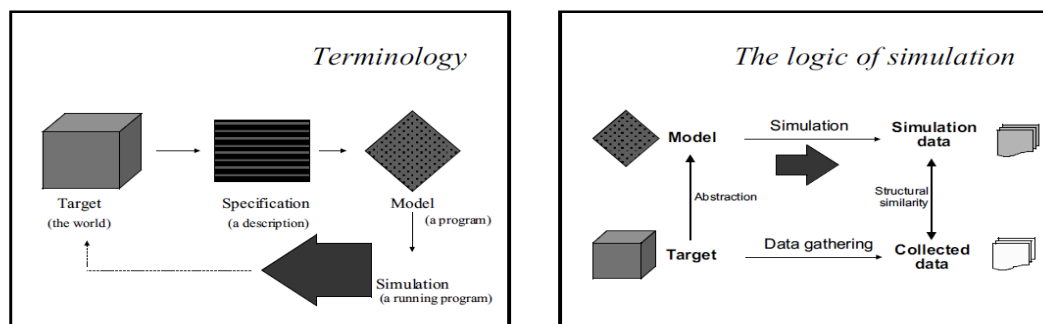


Figure 2.3: Logic of Simulation

2.2.2 Theory

The AB modelling theory is grounded on the base of three key ideas: agents, emergence, and complexity. In this theory, agents are goal oriented intelligent objects situated in space and time. They are interacting with one another based on predefined rules and behaviour. These agents and their operation and interactions can simulate a real world complexity over a sufficient period of time. The aim is to make assumptions which are as close as possible to the real world so that the model can regenerate the real world behaviour, thus the simulation results can be compared to the real system. The interesting and promising benefits of AB modelling, generally speaking, is defined as their potential for regenerating the equilibria of a system while the traditional analytic methods enable humans to characterise those equilibria.

Recent research on the modelling and simulation of complex adaptive systems by Muaz Niazi demonstrated the need for combining AB and complex network based models, suggesting a framework consisting of four levels of development using several examples of multidisciplinary case studies [Niazi, 2011]:

- **Complex Network Modelling** level for developing models using interaction data of various system components.
- **Exploratory AB Modelling** level for developing AB models for assessing the feasibility of further research.
- **Descriptive AB Modelling (DREAM)** level for developing descriptions of AB models by means of using templates and complex network-based models for the purpose of model comparison across scientific disciplines.
- **Validated AB Modelling** level using a Virtual Overlay Multiagent system (VOMAS) for the development of verified and validated models in a formal manner.

2.2.3 Applications

AB models are used by scientists, policy makers, managers and other professionals for the purposes of prediction, verification, analysis, and training [Davidsson et al., 2006]. AB modelling is widely being

applied in many areas including; business and organizations (e.g. manufacturing, consumer markets, supply chain [Fang et al., 2002] and insurance), infrastructure [Holmgren et al., 2008] (e.g. electric power markets, hydrogen economy and transportation), crowds [Henein and White, 2005] (human movement and evacuation modelling), society and culture (e.g. ancient civilizations [Kohler et al., 2005] and civil disobedience, social determinants of terrorism and organizational networks), military (e.g. command & control and force-on-force), biology [Folcik and Orosz, 2006, Carley, 2006, Huang et al., 2004] (e.g. ecology, animal group behaviour, cell behaviour and sub cellular molecular behaviour), helped policy-making in epidemiology [Auchincloss and Roux, 2008, Farmer and Foley, 2009], traffic control [Hadouaj et al., 2001], and economics (e.g. artificial financial markets and trade networks [LeBaron, 2002]). See [Davidsson et al., 2006] for a survey and analysis of applications of AB simulation (ABS).

2.3 Data Analysis

The main interest of most AB models is to simulate some real world phenomenon. A “good” model, generally speaking, is one that demonstrates the same behaviour, structural similarities (2.2.1), as the real world phenomenon [Davidsson et al., 2006]. However, these models do not use real world data for initializing the simulation parameters. Ignoring the collaboration of empirical data into the AB modelling might be the result of a generalised belief that the purpose of AB models experiments is to generate theoretical hypotheses in an empirical way, not to test these hypotheses [Prietula et al., 1998]. In social science, the AB models are commonly used as a laboratory tool which makes it possible to compensate for the unavoidable weakness of empirical and experimental knowledge [Boero and Squazzoni, 2005] and correlation between the theory and the observable evidence does not seem to be one of the vital priorities [Moss and Edmonds, 2005]. Thus, we witness a feeble linkage between theory, empirical reality and modelling from the early years [Merton, 1968, Schelling and al., 1998].

The importance of this issue in social science became noticeable because of various debates on social simulation [Eliasson and Taymaz, 2000], history-friendly models [Brenner and Murmann, 2003] and applied evolutionary economics [Pyka and Ahrweiler, 2004] in recent years. However, the matter of the empirical validation of models has been well-studied for many years in the ecological sciences [Carlson et al., 2002] and social insects studies [Jackson et al., 2004]. In 2005, Riccardo Boero and Flaminio Squazzoni argued how the quest for AB models internal verification has been the major centre of attention for many years while less consideration has been devoted to the quest for empirical calibration and validation and to the empirical extension of models within the social science community [Boero and Squazzoni, 2005].

To better explain the causes of ignoring empirical data and its consequences, next we focus on two matters: the classical view of AB modelling and model initialising.

2.3.1 The Classical View of Agent Based Modelling

As discussed in Section 2.2.1, the classical AB modelling methodology has been founded on the base of “*logic of simulation*” [Gilbert and Troitzsch, 2005] methodology, which consists of four key processes:

Abstraction, Simulations, Data gathering and Structural similarities as illustrated in Figure 2.3. As Figure 2.3 demonstrated, the target is being considered in two stages of the simulation. The first time is when the target is being processed and its abstraction is being used for forming the simulation design and structure. The second time is when the collected data from the target is being used for the validation process. Not surprisingly, the simulation is not aware of the starting conditions of different parameters as the collected data is not introduced to the model, thus the simulation data might not present any correlation to the collected data. This is why the classical AB modelling methodology does not support the incorporation of the real data by itself.

2.3.2 Model initializing

As explained in the classical approach 2.3.1, the model has been designed based on a high level of abstraction, therefore, the model may not incorporate empirical data which is precise in time and space. AB models are traditionally catered for by a standard distribution and this distribution is being used in several steps of the design: configuring the initial conditions of simulations, distributing objects spatially, and determining exogenous factors or aspects of the agents' behaviour [Hassan et al., 2010]. Although the simulation results obtained from such a model can provide insights or demonstrate some similarities to real world observations, they are not close enough to the corresponding targets because the choice of the initial conditions often has a direct impact on the dynamics of the model and simulated data. Hence, the model may not be feasible or reliable enough to be used in the real world. Using an abstract model and random values is advantageous as the model can be considered to be more general, being applicable to any circumstances within the bounds of the stochastic distributions used to obtain the parameter values.

Typically, the output of a model initialized by a uniform random distribution (commonly used distribution in AB modelling. e.g. [Barabasi et al., 2002] and [Newman et al., 2002]) is an aggregated mean of a series of simulations (each starts with a different random seed value). This is an appropriate technique to test the relationships among a set of parameters in a model [Hassan et al., 2008]. Though, it is not clear that the simulation result cannot be improved with other initial conditions, especially when precise data from a real system will be used for validation or one can argue that the uniform random distribution might not be suitable for all problems.

One solution is to use a specific distribution instead of uniform random distribution to get a better result in specific cases. For instance the probability distribution for the total distance covered in a random walk (biased or unbiased) tends toward a normal distribution [Dinov et al., 2008]. Another example is the usage of Poisson distribution for modelling earthquake frequency uncertainties in seismic hazard analysis [Greenhough and Main, 2008]. However, to know which distribution fits the observed data best, there has to be access to real data over a period of time, and in that case, it is better to use the real data directly, rather than abstracting them into a distribution.

A fundamental problem of probability distributions is that while they are good at describing aggregate behaviours [Hassan et al., 2010], they are incapable of providing reasons that cause individual behaviour or events. Moreover, the comparison of the mean of multiple runs with one observation of the target cannot be reliable as the output may not have a stable mean or the observed target might be

an outlier. An example of observed targets which have non-stable means (Figure 2.4) can be found in phenomena that exhibit a power-law²⁰ distribution such as the proportion of scientific papers that receive N citations [Easley and Kleinberg, 2010].

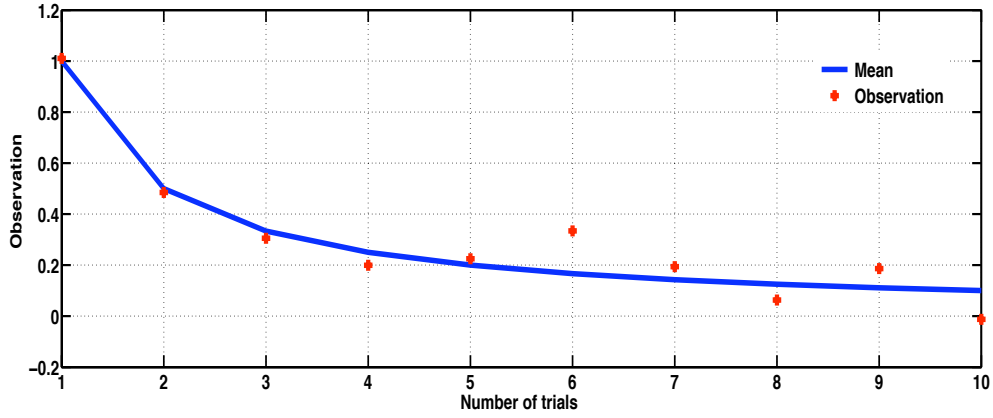


Figure 2.4: Illustration of an observed target which has non stable means

An example of an observed target with an outlier characteristic can be found in phenomena with stochastic²¹ behaviour such as stock²² prices [Aguilera et al., 1999]. This becomes a problem as the target with stochastic behaviour can be far from the mean and, therefore, an outlier (Figure 2.5). In other words, if the observed value happens to be an outlier, the discrepancy between the simulation result and corresponding targets could be very large [Hassan et al., 2008]. In contrast, by taking into account the real world data for initializing the simulation parameters, we are much more likely to decrease the discrepancy between the model and the observed data (assuming the model is an accurate description of the world) as the data will, to some degree, enforce the model to follow the direction of the real world.

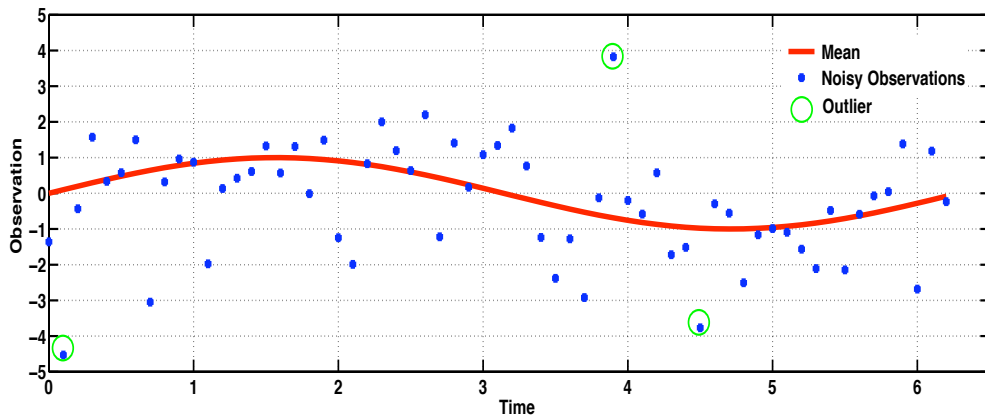


Figure 2.5: Illustration of an observed target which has outlier characteristic

²⁰A power law is a mathematical relationship between two quantities. When the frequency of an event varies as a power of some attribute of that event (e.g. its size or popularity), the frequency is said to follow a power-law.

²¹Refers to systems whose behaviour is intrinsically non-deterministic in that a system's subsequent state is determined both by predictable actions and by a random element.

²²The capital stock (or stock) of a business entity represents the original capital paid into or invested in the business by its founders.

In recent years, successful examples of the introduction of empirical data into AB models for tackling the issue of random initialisation have been investigated. Macro simulation technique [Orcutt, 1990, Anon, 2000], Anasazi culture change [Dean et al., 2000], pedestrian modelling [Batty, 2001], simulations of the electricity market [Nicolaisen et al., 2001], water demand models [Edmonds and Moss, 2005, Galan et al., 2007], social science [Boero and Squazzoni, 2005], and analytical sociology [Hedstrom, 2005] are prominent examples where the simulation results are considerably improved by using empirical data instead of random initialisation. A study of the Eurovision Song Contest²³ by Derek Gatherer [Gatherer, 2006] is another successful example where it exhibits how the AB modelling output can be improved by incorporating the real world data. In this paper, Gatherer introduced some empirical information of the observed target such as distance between countries and similarity of their cultures into the AB modelling with the hypothesis that results of the Eurovision contest would approximate to random over a sufficient long period of time. The simulation results ruled out the random voting schema by presenting how the simulation results improve when empirical data are used instead of random initialisation. Based on the simulation behaviour, if a country is closer or has a similar culture people are more likely to vote for it. However, the randomisation can still be tracked in different elements of these models.

2.3.3 ACE and Empirical Data

Not paying attention to the real world data within the ACE community is a consequence of two major challenges: introducing empirical data into AB modelling and the complexity of economics. Previously, we discussed the challenge of introducing empirical data into AB modelling by highlighting the random initialisation problems and explaining how the classical AB modelling methodology (logic of simulation method) lacks the fundamental concepts of employing real world data for initialising the model parameters and conditions. This section is focused on explaining how the complexity of economics affected the ACE community. In economics, as mentioned in Section 2.5, AB modelling researchers never attempted to build an AB model that can be used in the real world applications such as time series forecasts. This is due to two threads of ACE modelling: hypothetical architecture of ACE and real world data challenges.

Hypothetical architecture of ACE. The world economy is a complex system with a very complex relationship between its components. As discussed, the AB modelling provides an opportunity for investigators by letting them simplify the phenomenon complexity through an abstraction process. As a result of this simplification process alongside employing non empirical data, the utility of AB modelling for real world application is deeply limited. To date, most of AB models of economics are built on the conceptual level and have not been parameterised or initialised by real world data, however empirical data is used for validation purposes. These models take into account the qualitative properties of the real world economy, but not matching it in any detail has led to impracticality and infeasibility in the existing models.

²³The Eurovision Song Contest is an annual competition held among active member countries of the European Broadcasting Union. Each member country submits a song to be performed on live television and then casts votes for the other countries' songs to determine the most popular song in the competition.

Real world data challenges. Real world data is a priceless element of any investigation as it provides a bridge between theoretical and practical research. In the field of economics and finance, data acquisition and availability has been a major problem for academic practitioners for many years. This is due to the secretive nature of the financial industry, non transparency of some activities, and non-availability or limitation of data for some specific markets. Further, similar to many other industries such as marketing and telecommunication, but more seriously, data (or information) means money in finance. The financial industry relies on on-time and vital information for making money, thus this information has been commercialised and free of charge but access is restricted in many cases. Moreover, some participants might not publish their business details at all or the information might be kept secret for a period of time to make sure other participants cannot take advantage of it ²⁴.

2.3.4 Discussion

Feeding ACE models with real world data can contribute to obtaining simulation results that are closer to the real world corresponding target hence providing more viable models. In addition to the challenge of obtaining the real world data and incorporating it into AB models, tackling the problem of limited data length for long simulation time is still a challenge to be addressed. ACE provides an opportunity to study the behaviour of individuals as well as the market over time. Although the AB modelling has no restriction over the simulation time, incorporating the real world data into AB models will pose the restriction due to real world data length limitation. For instance, imagine we are investigating the behaviour of a company over time and of interest is to study all the possible future scenarios. A simple approach is to model a company's fundamental properties such as assets, liabilities, etc and initialize them by obtaining information from a real company, running the experiments under different conditions, and examining the possible outcomes. However, this can be only realistic if we assume that the company is completely independent and no other external factors affect it over time. There might be an external trend which the company follows. For modelling this trend we need to introduce the time series of this trend into the AB model. The critical situation appears if it is decided to run the simulation for a longer period of time than the time for which we have the trend data available. This problem limits the practicability of AB models for studying future scenarios while working with the real world data. Generating data which has the characteristics of real world data is a viable solution to tackle the problem of real world data length limitation, thus providing an unlimited realistic data feed for AB models.

2.4 CDS Pricing Techniques

As discussed earlier, default risk is expressed through the CDS spread. The CDS spread for a company is negatively related to its credit rating: the worse the credit rating, the higher the CDS spread [Hull et al., 2004]. Pricing of this CDS spread is a challenging open problem that requires a quantitative and qualitative approach involving estimations of default, timing of default and balance sheet value fluctuations (see: [Hull and White, 2000, Joro et al., 2004, Anon, 2008]). There are two grounded theories

²⁴Private communication with industry experts.

referred to which are used to price a CDS contract: *Probability Model* and *No-Arbitrage Model*. The probability model, takes the present value of a series of cashflows weighted by their probability of non-default. This approach suggests that CDSs should trade at a considerably lower spread than corporate bonds [Nakisa, 2011]. The no-arbitrage model, proposed by Darrell Duffie [Duffie, 1999], but also by John Hull and White [Hull and White, 2000, Hull et al., 2004], assumes that the arbitrage is not risk free. These models are discussed in more detail below.

2.4.1 Probability Model

Under the probability model, the CDS pricing model takes four inputs:

- Issue premium (c), which is a quarterly payment from CDS buyer to CDS seller.
- Recovery rate (R), which is the proportion of notional repaid if the default event occurs.
- Credit curve for the reference entity, which is the graphical representation of default probability of an entity over various time horizons.
- LIBOR curve, which is a graphical representation of various maturities of the London Interbank Offered Rate (LIBOR). This is the short-term floating rate at which large banks with high credit ratings lend to each other. See Section 2.4.2 for the application of LIBOR curve.

In the case of no default, the price of CDS would be the sum of the discounted premium payments (see Table 2.2). However, the CDS pricing models have to consider the probability of the default occurring at some time between the effective date (starting date of the contract) and maturity date (expiry date of the contract) of the CDS contract.

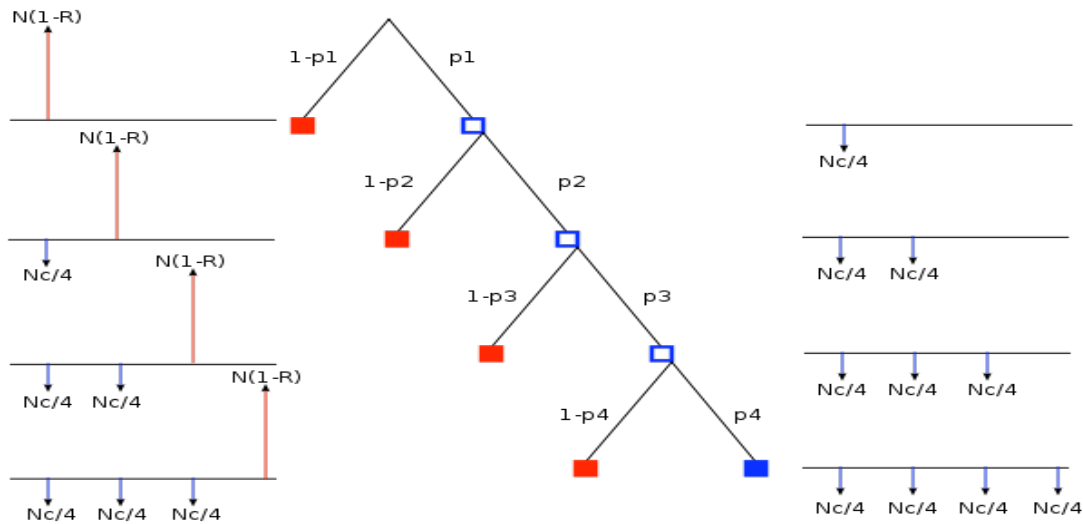


Figure 2.6: Cds Pricing Tree (source: [Nakisa, 2011]): a payment date in “no default” case is illustrated by a white square with blue borders and the “default” case by a red square. The contract expiration date is presented by a blue square. p_1 refers to the probability of no default at time T_1 , while $1 - p_1$ refers to the probability of default at time T_1 .

The probability model can be illustrated with an example from Nima Nakisa [Nakisa, 2011] (a global asset allocation strategist at an investment bank.). A one year CDS contract has an effective date T_0 , nominal N , issue premium c , and four quarterly premium payments occurring at times T_1, T_2, T_3 and T_4 (Table 2.2 presents the payment dates by square symbols.). The quarterly premium payments then can be calculated as $\frac{Nc}{4}$. For the sake of simplicity it is assumed that the defaults only happen on one of the payment dates. Therefore, the contract can be terminated in two scenarios and five ways: the contract expires by reaching the end of its term, or there is a default event which terminates the contract. In the case of expiration, the four premium payments are made and the contract survives until the expiry date. In the case of a default event, there are four times T_1, T_2, T_3 and T_4 at which the default can happen and the compensation payment should be made. The actual price of a CDS is simply the present value of the five payoffs multiplied by their probability of occurring. So the first step is to calculate the probability of each possible outcome and the present value of each payoff. Figure 2.6 illustrates the cash flows of the CDS contract for this example in a tree diagram.

At each payment date either the contract has a default event shown in red or it survives without a default being triggered and is shown in blue. If the default occurs, the contract ends with a payment of $N(1 - R)$, where R is the recovery rate. If the contract survives, a premium payment of $\frac{Nc}{4}$ is made. At either side of the diagram are the cashflows up to that time with premium payments in blue and default payments in red. If the contract is terminated, the square is shown with solid shading.

The surviving probability over the interval T_{i-1} to T_i without a default payment is P_i . Therefore, the probability of a default being triggered is $1 - P_i$. Letting the σ_i to be the discount factor²⁵, the present value can be calculated as below.

Table 2.2: CDS Present Value and Probability Calculation

Description	Premium Payment PV	Default Payment PV	Probability
Default at time T_1	0	$N(1 - R)\sigma_1$	$1 - P_1$
Default at time T_2	$-\frac{Nc}{4}\sigma_1$	$N(1 - R)\sigma_2$	$P_1(1 - P_2)$
Default at time T_3	$-\frac{Nc}{4}(\sigma_1 + \sigma_2)$	$N(1 - R)\sigma_3$	$P_1P_2(1 - P_3)$
Default at time T_4	$-\frac{Nc}{4}(\sigma_1 + \sigma_2 + \sigma_3)$	$N(1 - R)\sigma_4$	$P_1P_2P_3(1 - P_4)$
No default	$-\frac{Nc}{4}(\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4)$	0	$P_1P_2P_3P_4$

The probabilities P_1, P_2, P_3, P_4 can be calculated using the credit spread curve. The probability of no default occurring over a time period from T to $T + \Delta T$ decays exponentially with a time-constant determined by the credit spread, or mathematically $P = \exp(\frac{-s(T)\Delta T}{1-R})$ where $s(T)$ is the credit spread zero curve at time T . The riskier the reference entity the greater the spread and the more rapidly the survival probability decays with time. To calculate the total present value of the CDS we multiply the probability of each outcome by its present value to give

²⁵The discount factor, DF(T), is the factor by which a future cash flow must be multiplied in order to obtain the present value.

$$\begin{aligned}
PV &= (1 - P_1)N(1 - R)\sigma_1 \\
&+ P_1(1 - P_2)[N(1 - R)\sigma_2 - \frac{Nc}{4}\sigma_1] \\
&+ P_1P_2(1 - P_3)[N(1 - R)\sigma_3 - \frac{Nc}{4}(\sigma_1 + \sigma_2)] \\
&+ P_1P_2P_3(1 - P_4)[N(1 - R)\sigma_4 - \frac{Nc}{4}(\sigma_1 + \sigma_2 + \sigma_3)] \\
&- P_1P_2P_3P_4(\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4)\frac{Nc}{4}
\end{aligned} \tag{2.8}$$

2.4.2 No-arbitrage Model

The no-arbitrage model assumes that the arbitrage is risk free. Duffie [Duffie, 1999] uses the LIBOR as the risk free rate, whereas Hull and White use US Treasuries as the risk free rate. Both analyses make simplifying assumptions (such as the assumption that there is zero cost of unwinding the fixed leg of the swap if a default event occurs), which may invalidate the no-arbitrage assumption. However the Duffie approach is frequently used by the market to determine theoretical prices. In theory CDS spreads should be closely related to bond yield spreads [Hull et al., 2004]. The Duffie approach provides a method to evaluate the correct pricing of the CDS spread through the simple relationship.

$$S = Y - R \tag{2.9}$$

Where S is the n -year CDS spread, Y is the yield on an n -year par yield bond issued by a reference entity (e.g. risky company), and R is a yield on an n -year par yield “riskless bond” (e.g. interest rate). This relationship is observed in the market and if it breaks down significantly, traders will buy and sell the instruments to return the relationship close to parity. But observing the CDS spread, debt return and risk free rate in the financial market shows that this relationship does not hold exactly. An example of this observation is illustrated in figure 2.7. The reason for the gap between the Duffie theory and market data is the cost of arbitrage or what is also known as the arbitrage channel [Francis et al., 2003]. If S is greater than $Y - R$, an arbitrageur will find it profitable to buy a riskless bond, short a corporate bond and sell the CDS. If S is less than $Y - R$, the arbitrageur will find it profitable to buy a corporate bond, buy the CDS and short a riskless bond [Hull et al., 2004].

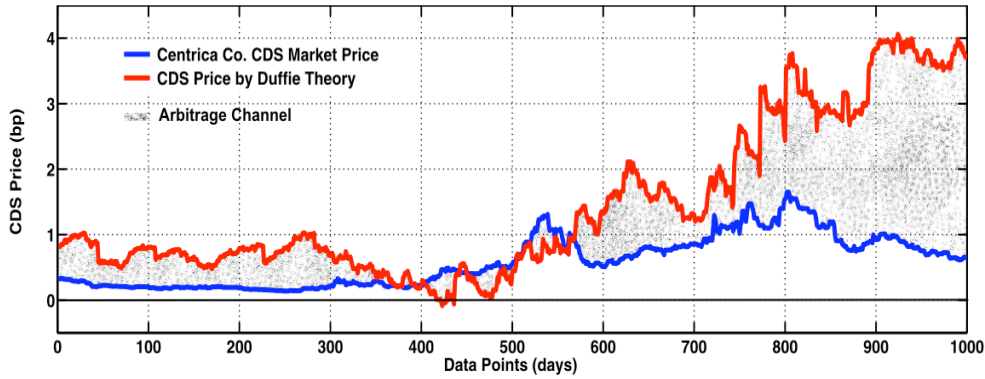


Figure 2.7: Duffie Theory Vs. Market Data: the CDS price is collected from Centrica company for the period of 1000 days.

The arbitrage cost comes from the range of market mechanics to borrow, sell and buy instruments to profit from the CDS spread, inaccurately estimating the risk of a default event [Hull and White, 2000].

2.4.3 Discussion

As explained, the CDS spread can be seen as an indicator of a company's health. If investors (e.g. protection buyers) are worried about a firm's credit quality they can buy protection thus pushing up CDS spreads on that name in the market. Consequently, the company's stock price will be pushed down based on the fear of the company's default. The CDS spread plays a key role in the CDS market due to the opportunity that it provides for the parties to either gain profit or reduce the loss and outside of CDS market due to the impacts that it has on the rest of the market (e.g. stock market). Here, there will be clarification of the sufficiency of the two theoretical CDS pricing approaches, discussed in this section, in respect to the objectives of this thesis.

The ultimate goal of providing a feasible CDS pricing technique, in this research, is to supply our "agents" with a pricing technique which they can use to value the CDS contracts. Because of the simulation method which is chosen, AB modelling, the compatibility of the pricing technique with the AB model needs to be ensured. There are two primary concerns associated with this matter as follows:

- The method should not require sophisticated information to avoid the unnecessary complexity. Section 2.2 discussed how important is to keep the model simple (K.I.S.S. principle). By introducing the complex objects to the model at the early stage, it is probable that the result will be as incomprehensible as the real world.
- The method should provide the best price (as close as possible to the real market price) using the available information. In many cases, the available information is limited. Therefore, it is important to have a method which has a degree of flexibility in regards to its input parameters while the output remains in a acceptable range for the specified purpose.

Due to concerns about pricing techniques, the sufficiency of both pricing technique (probability mode and no-arbitrage model), discussed in this section, are arguable. The probability model requires information such as issue premium, recovery rate, credit curve, and LIBOR curve. However, the recovery rate is not known for each individual contract due to the unavailability of data. Moreover, estimating the default probability of an entity over a period time is a quantitative, qualitative and complex task which is outside the scope of this thesis.

In contrast to the probability approach, no-arbitrage approach provides a relatively simple method for pricing CDS contracts. The no arbitrage approach requires only bond yield information and interest rate (this information are available in the market and can be obtained from different sources such as Reuters) to calculate the contract price. However, the calculated price is far from the real market price due to the arbitrage channel, see Figure 2.7, thus the method is not feasible in practice.

In the real world, based on private communication with industry experts, CDS pricing techniques are either not observable or not available for the public. This is due to the financial benefit that these techniques have at the time for their institutions. However, it might be possible to access these techniques

(if their institutions publish them) when they are no longer efficient. In this these thesis, we investigate whether it is possible to estimate CDS pricing techniques from the observable market price. Chapter 5 will present our investigation in detail.

2.5 Agent Based Computational Economics

As mentioned in Chapter 1, ACE is a branch of AB modelling concerned with computational study of economies modelled as evolving systems of autonomous interacting agents under controlled experimental conditions [Tsfatsion, 2003], with an officially designated special interest group (SIG) of the Society for Computational Economics²⁶. ACE lies in the paradigm of Complex Adaptive Systems²⁷ [Arthur, 1994, Tsfatsion, 2003]. The ACE field has been developed over the emergent interference of three research streams: computer science, cognitive science and evolutionary economics [Anon, 2006]. It has a similarity to, and overlap with, game theory for modelling social interactions [Shoham, 2008, Roth2, 2002], however, a number of differences have been noted by practitioners [Tsfatsion, 2006]. ACE models start from initial conditions specified by the modeller, the computational economy evolves over time as its constituent agents repeatedly interact with each other (the agents may learn from their interactions). In these respects, ACE has been characterized as a bottom-up culture-dish approach to the study of economic systems [Tsfatsion, 2002]. The current ACE research objectives can be divided into four strands [Anon, 2006, Tsfatsion, 2006, Tsfatsion, 2003]:

- **Empirical understanding.** The aim of this research strand is to understand the reasons behind the evolution and persistence of observed regularities such as trade networks, socially accepted monies, market protocols, business cycles, and the common adoption of technological innovations.
- **Normative understanding.** Here, researchers are interested in utilizing ACE for the discovery of good economic designs. The key question to be addressed is whether designs proposed for economic policies, institutions, or processes will result in socially desirable system performance over time.
- **Qualitative insight and theory generation.** Within this community, the goal is to understand how ACE models can be used to gain a better understanding of economic systems through a better understanding of their full range of potential behaviours over time.
- **Methodological advancement.** ACE researchers are exploring a variety of ways such as the application of methodological principles to the practical development of programming, visualisation, and validation tools to provide the best techniques and tools for studying the economic systems through systematic computational experiments.

An overview of ACE is provided by Leigh Tsfatsion [Tsfatsion, 2002] where the author outlines the main objectives and defining characteristics of the ACE methodology, and discusses similarities

²⁶See SIG website for more information: <http://comp-econ.org/>

²⁷Complex adaptive systems are a kind of complex systems, often defined as a “complex macroscopic collection” of relatively similar and partially connected micro-structures, formed in order to adapt to the changing environment, and increase its survivability as a macro-structure [Mitleton-Kelly, 2003].

and distinctions between ACE and artificial life research. Open questions and directions for future ACE research are also considered. The study concludes with a discussion of the potential benefits associated with ACE modelling, as well as some potential difficulties. Table 2.3 summarises the research undergone so far in each category [Tsfatsion, 2001b, Tsfatsion, 2001c, Deissenberg et al., 2008, Heckbert et al., 2010, Tsfatsion, 2006].

Current ACE²⁸ research has been categorised into seven areas:

1. **Learning and the embodied mind:** ACE is taking advantage of a broad range of representations for the learning processes of computational agents such as reinforcement learning algorithms, neural networks, genetic algorithms, genetic programming, and a variety of other evolutionary algorithms and inductive learning[Brenner, 2006]. Many of these learning representations were originally developed with global optimality objectives in mind, so caution must be used in applying them to economic processes[Tsfatsion, 2003].
2. **Evolution of behavioural norms:** Behavioural economics uses facts, models, and methods from other sciences such as psychology, sociology, anthropology, and biology to establish descriptively accurate findings about human cognitive ability and social interaction and to explore the implications of these findings for economic behaviour[Anon, 2003b]. The study of the evolution of behavioural norms helps to understand how patterned social behaviour can arise as the unintended consequence of repeated local interactions among agents following simple behavioural rules[Schelling, 1978].
3. **Bottom-up modelling of market processes:** One of the most active areas of ACE research is the self-organising capabilities of specific types of market processes[Tsfatsion, 2003]. According to [Tsfatsion, 2001a] and [Tsfatsion, 2001c], several types of markets have been investigated by ACE investigators including: financial; electricity; labour; retail; business-to-business; natural resource; entertainment; and automated Internet exchange systems.
4. **Formation of economic networks:** A network is a collection of entities together with a specified pattern of relationships among these entities. Three main tools have been used for the quantitative study of networks: graph theory; statistical and probability theory; and algebraic models. ACE investigators interested in interaction networks have generally focused on imperfectly competitive markets involving strategic interaction among a small numbers of buyers and sellers. In such markets the deliberate choice of partners through learning and the development of trusted relationships can greatly influence the form of the interaction networks that arise and persist[Tsfatsion, 2003].
5. **Modelling of organisations:** The AB approach views an organisation as a collection of agents, interacting with one another in their pursuit of assigned tasks. Therefore, the organisation performance is determined by the formal and informal structures of interactions among agents, which define the lines of communication, allocation of information processing tasks, distribution of

²⁸See <http://www.econ.iastate.edu/tesfatsi/ace.htm> for extensive resources related to the ACE methodology.

decision-making authorities, and the provision of incentives. The common research questions of this literature are: 1) What are the determinants of organisational behaviour and performance? 2) How does organisational structure influence performance? 3) How do the skills and traits of agents matter and how do they interact with structure? 4) How do the characteristics of the environment – including its stability, complexity, and competitiveness – influence the appropriate allocation of authority and information? 5) How is the behaviour and performance influenced when an organisation is co-evolving with other organisations from which it can learn? 6) Can an organisation evolve its way to a better structure?[Anon, 2006]

6. **Design of computational agents for automated markets:** In addition to saving labour time, automated contracting through computational agents can increase search efficiency in certain problem applications. A large number of researchers are now involved in the design of computational agents for automated markets and much of the work has focused on implementation, enforcement and security issues[Tesfatsion, 2003].
7. **Parallel experiments with real and computational agents:** This research area examines the relationship between AB modelling and economic decision-making experiments with human subjects. Both approaches exploit controlled "laboratory" conditions as a means of isolating the sources of aggregate phenomena. Research findings from laboratory studies of human subject behaviour have inspired studies using artificial agents in "computational laboratories" and vice versa. In certain cases, both methods have been used to examine the same phenomenon[Anon, 2006].

2.5.1 ACE Trends

AB economic research (Table 2.3 highlights some of these studies) undergone to date has inclined to concentrate on two objectives: *qualitative understanding of interactions* and *replicating stylised facts*.

Qualitative understanding of interactions. This research stream is focused on understanding the actors, their interactions, their restrictions, and envisioning the possible outcomes by carefully modelling the sufficient details of an AB system. Such a model can assist in understanding the underlying logic of the economy and its components, to identify positive and negative feedbacks, and to study the agent behaviours in different scenarios, leading to insight into the consequences of interactions. Although this approach does not require data, it provides useful qualitative information. Most economic AB models to date have been constructed and utilised at this level. However, in many cases the components of the model and their results have been criticised as being ad hoc.

Replicating stylized facts. This research stream is centred on replicating stylised facts such as distributions for income, firm size, price returns, or functional relationships such as the Phillips curve relating price movements to unemployment. This replication can be done at various levels of accuracy. For instance, one can aim to replicate a probability distribution for the purpose of qualitative properties, correct functional form, correct quantitative properties, or fully matching all the moments of the distribution, including the correct location and scale.

Table 2.3: ACE Research Areas

Research Area	Investigators
Learning and the embodied mind	Koza (1992), Holland (1992), Sutton et al. (1998), Chattoe (1998), Dawid (1999), Terna (2000), Bell (2001), Picton (2001), Chen (2001), Luna(2002).
Evolution of behavioral norms	Schelling (1987), Holland (1995), Arifovic et al. (1995), Epstein et al. (1996), Riolo (1997), Axtell et al.(1999), Hoffmann (2000), Epstein (2001), Chen et al. (2011).
Bottom-up modeling of market processes	Financial Markets studies: Lux (1998), Farmer and Lo (1999), Lux et al.(2000), LeBaron (2000), Howitt and Clower (2000), Tay and Linn (2001), Izumi and Ueda (2001), Chen and Yeh (2001), Hommes (2001). Electricity Auction Markets: Klemperer (2000), Klemperer (2001), Bower and Bunn (2001), Bunn and Oliveira (2001) Nicolaisen et al. (2001), Twomey et al. (2002), Tesfatsion (2002), Lye et al. (2012), Tay (2013).
Formation of economic networks	Albin and Foley (1992), Vriend (1995), Weisbuch et al.(1995), Kirman (1997), Tesfatsion (1997a and 1997b), Watts and Strogatz (1998), Tesfatsion (1998), Wilhite (2001), Tassier and Menczer (2001) Kirman and Vriend (2001), Klos and Nooteboom (2001) Rouchier et al. (2001), De Vany and Lee (2001), Leigh Tesfatsion (2006), Deissenberg et al. (2008), Andrew Reeson et al. (2010).
Modeling of organizations	Prietula et al. (1998), Van Zandt (1998), Joslyn et al. (2000), Chang et al. (2005), Dosi et al (2013).
Design of computational agents for automated markets	Andersson and Sandholm (2001), Kephart (2002).
Parallel experiments with real and computational agents	Miller and Andreoni (1990), Marimon et al. (1990), Arthur (1991), Andreoni and Miller (1995), Arifovic (1994 and 1996), Chan et al. (1999), Duffy (2001), Pingle and Tesfatsion (2001).

2.5.2 ACE Applications

In the past decade, the AB financial market has become one of the most active and popular research areas of ACE as it successfully managed to provide insights and possible explanations for a range of observed happenings in financial markets such as modelling and explaining the stylised facts of financial markets [Hommes, 2002], studying the social and economic interaction of speculators in a securities or foreign exchange market [Lux, 1998], and explaining the finding of clustered volatility and ARCH effects in financial data [Lux and Marchesi, 2000]. A study by Nicolaisen et al. [Chany et al., 1999] modelled a repeated double-auction market where it successfully replicated several findings of human-based experimental markets, however, an intriguing difference between AB and human-based experiments have been found. Incorporating evolutionary techniques such as genetic programming with AB economic models became fashionable in the 1990s when Shu-Heng Chen et al. utilized genetic programming in AB model of stock market [Chen and Yeh, 1999, Chen and Yeh, 2001]. A critical review of the fundamental issues of genetic programming applications to ACE is provided by Chen where she argues that genetic programming is not well grounded in considerations of human behaviour and the issues of primitives, semantics, genetic operators, and architecture need to be investigated further [Chen, 2001].

Some small portions of the economy have been successfully modelled by AB systems. For instance, Blake LeBaron modelled the financial market and presented a reasonable explanation for bub-

bles²⁹ and crashes, replicating crises and crashes that never appear in traditional economic models [Chany et al., 1999, LeBaron, 2011, LeBaron, 2002]. Rob Axtell has devised firm dynamics models that simulate how companies grow and decline as workers move between them. These reproduce the power-law distribution of company size that one sees in real life: a very few large firms, and a vast number of very small ones with only one or two employees [Farmer and Foley, 2009].

In 2000, Blake LeBaron [LeBaron, 2000] surveyed some of the earliest AB financial market models in detail including the highly influential Santa Fe artificial stock market [Arthur et al., 1997] and an AB approach for the modelling of foreign exchange markets proposed by Izumi and Ueda [Izumi and Ueda, 2001] to overcome the challenges of predictive powers using conventional modelling approaches. The research stream of AB macro-economics has concentrated on building computational laboratories capable of jointly replicating as many stylised facts at the cross-sectional and macro levels. Micro simulations, pioneering work of Guy Orcutt [Orcutt, 1990, Anon, 2000], was the first attempt to generate emergent macro economics, however, the scope of their results was restricted due to poor computational power. Micro simulation investigates the behaviour of individuals over time, where the individuals are initialised with real world data (e.g. where the data is derived from a sample survey). However, individuals are isolated and the interactions between them are not modelled. The ASPEN model by Pryor et al. [Pryor et al., 1996, Basu et al., 1998] was the first large-scale AB macro model of the US economy to be implemented as a super computing application, but with little empirical relevance. Delli Gatti et al. [Gatti et al., 2005] developed a model, which is capable of replicating power-law firms' size distributions, along with a several financial stylised facts. The model simulated a banking sector lending to firms with heterogeneous balance-sheet soundness. Edoardo Gaffeo et al. [Gaffeo et al., 2008] also reproduced the size distribution and business cycle stylised facts. The model suggests relevant but distinct mechanisms motivating the emergence of stylised facts, however, it lacks stock-flow consistency, thus it shares the unrealistic feature of open systems.

2.5.3 Most Recent Relevant Work

Despite the important role of CDS market in 2007-2010 credit crisis and also the promising potential of AB modelling, there has been no attempt to investigate this market using AB modelling to date. However, as explained in Chapter 1, both academia and the financial industry have started to realise the potential of AB modelling since 2010 leading to an interest in novel research projects. There are three major works in recent years which share similarities with this thesis. These works will be introduced and discussed in detail in the next section.

1 - EURACE Project

EURACE is an AB Software Platform for European Economic Policy Design with Heterogeneous Interacting Agents. This project was carried out from 2006 to 2009 in the framework of the European 6th framework programme by a consortium of economists and computer scientists including nine uni-

²⁹A theory that security prices rise above their true value and will continue to do so until prices go into freefall and the bubble bursts.

versities from the UK, Italy, Germany, France, Turkey, and the USA³⁰. Based on the project objectives, EURACE aims to construct an exhaustive agent-based model of the European economy, populated by a very large number of sophisticated, autonomous agents. It offers a unique opportunity for studying the empirically observed but theoretically poorly understood link between the real and the financial sphere of a modern European economy [Deissenberg et al., 2008].

As reviewed in Section 2.5, most of the existing ACE models, cover only a single industry, one restricted geographical area, or a unique market, and involve relatively small populations of agents. However, the EURACE project took a massive step forward by developing a model that integrates many of the major markets considered in quantitative macroeconomic modelling (e.g. consumer goods, investment goods, labour, credit and finance) [van der Hoog et al., 2008]. However, it still lacks key features that played an essential role in the recent crisis, such as the housing market or a well-developed banking sector which effect the accuracy of the final results.

From the perspective of this thesis, the EURACE project like many previous ACE models, has limited potential to be deployed in the real economy due to their hypothetical features. These models have been built on hypothetical economies considering some of the properties of the real economy at a qualitative level, but not matching it in any detail. To tackle this problem and develop a model which can be used to study the real world economy, it must be possible to initialise the model in a state corresponding to a real economy and be able to feed the model real data or a realistic time series.

2 - Microstructure Dynamics and Agent-Based Financial Markets

Michael Kampouridis, Shu-Heng Chen, and Edward Tsang made an investigation of whether the financial markets' behaviour is non-stationary or not in 2011 [Chen et al., 2011]. They investigated the fraction dynamics of trading strategies by employing Genetic Programming (GP) as an inference engine for trading rules, and Self-Organizing Maps (SOM)³¹ for clustering these rules into trading strategy types. Their initial work appeared unrealistic due to their assumptions that strategy types remain the same over time. However, they addressed this issue in their follow-up paper and extended their model which enables the investigation of market behaviour dynamics [Kampouridis et al., 2011]. Their result showed that old strategies cannot successfully be re-applied to future periods which means the markets' behaviour is dynamic. The authors claimed that this is due to the continuous change of market conditions over time. Thus, although the old strategies can demonstrate some relatively good performance they cannot be as successful as they initially were.

This thesis is concerned with the part of this work that GP is used as a trading rules inference engine. The main assumption of this work is that traders' behaviour (e.g. trading strategies) is either not observable or not available. This is a valid assumption when dealing with financial markets due to information security. For instance, in the case of CDS pricing 2.4, the available CDS pricing techniques

³⁰See <http://www.eurace.org/> for more information on the EURACE project.

³¹A self-organizing map (SOM) or self-organizing feature map (SOFM) is a type of artificial neural network (ANN) that is trained using unsupervised learning to produce a low-dimensional (typically two-dimensional), discretized representation of the input space of the training samples, called a map. See http://en.wikipedia.org/wiki/Self-organizing_map for more information and references.

are either not practically sufficient or they are too complex to be used by an AB model. Chapter 5 explains how this challenge is addressed by using a form of GP to price the CDS contracts reasonably by using observable market information.

As the authors explained [Kampouridis et al., 2011], they use GP to approximately infer the trading strategies from the observable market information. Each individual of the population is defined as a Genetic Decision Tree (GDT) where a GDT is a market-timing strategy (such as a financial advisor) combined by GP and its recommendation is to buy (1) or not-buy (0). Figure 2.8 illustrates an example of a GDT.

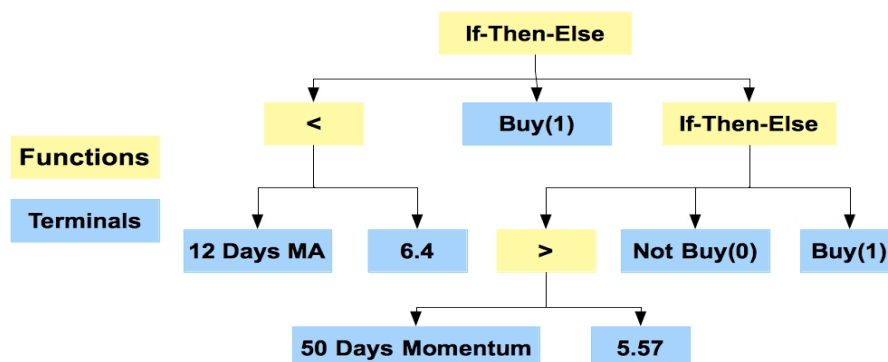


Figure 2.8: An Example of Genetic Decision Trees

Given a set of historical data and the fitness function (See [Chen et al., 2011, Kampouridis et al., 2011] for more details.), GP is then applied to evolve market-timing strategies. After evolving for a number of generations, the last generation (survivors) is, presumably, a population of financial agents whose market-timing strategies are financially successful. Although the result of this research seems promising, as the authors mentioned, the derived conclusions are based on a single dataset and for a general universal conclusion a more comprehensive dataset is required.

Furthermore, the certain aspects of the approach such as the used dataset and GP settings are unclear and not specified in detail thus it is difficult to reproduce their work and corroborate their results. Moreover, it would be interesting to know the computational time of the evolved solutions as the practicability of this approach for day to day trading is not only dependent on the best solution but how long it takes to achieve this solution.

3 - CRISIS Project

Complexity Research Initiative for Systemic Instabilities (CRISIS) project is an ambitious three years project starting in November 2011 in the framework of the European 7th framework program. This is a collaborative project carried out by a consortium of 12 leading academic and private sector institutions from Italy, Germany, Netherlands, Spain, Austria, United Kingdom, Hungary, and France³².

The project aims to deliver a macroeconomic AB model of the EU with a sophisticated financial system and a user-friendly graphical interface and web-based gaming mode. The core element of the

³²See <http://www.crisis-economics.eu/home> for more information on the CRISIS project.

project is defined as a pair of coupled AB models of the European economy (financial system and macro-economy). Furthermore, the European model is supposed to be coupled to a corresponding American model developed in an independent project. These models will be carefully calibrated using a comprehensive data set and will be built around the available data, thus the resulting model can be placed in the existing state of the real economy at any given point in time, and used to simulate that economy going forward. In addition, laboratory experiments with human subjects will be performed for the decision making components. Moreover, a sophisticated graphical user interface with open-source software will be delivered as well. This model aims to help researchers to better understand the combined European and American economies, freed from the constraint of unrealistic assumptions such as perfect rationality and representative agents.

The latest workshop of the project, October 2013, called “CRISIS at work: Explaining and Managing Financial-Real Interlinkages focused on issues linked to the process of integration between the macro and financial parts of the project. The partners discussed the project’s progress, the various difficulties encountered in each unit’s research work and guidelines for future activities were agreed. Today (December 2013), the CRISIS project is in the last quarter of its lifetime. Based on the project website, various article have been published explaining different modules and work packages of the project, however these modules are not integrated completely thus the final outcome of the project is yet to be announced.

2.5.4 Discussion

As illustrated by the models described above, all the AB models built so far in economics tend to be for specialised purposes and they only model a given piece of the economy such as a financial market (SFI Stock market, Brock and Hommes, Lebaron, Lux, Farmer, Thurner et al), credit markets (Delli Gatti and Gallegati), or firm size (Axtell) without integrating the pieces together into a comprehensive model. However, this is usually the common and only way that anything can be modelled - taking a smaller subset of the whole. Moreover, despite the poor forecasts of econometric and DSGE models (e.g. forecasting a few quarters ahead), none of the AB economic models to date has been attempted to make forecasts of time series. In contrast to the economics field, in other fields AB models are utilized successfully in epidemiology for modelling the spread of diseases in detail, and to predict the unfolding of a crisis and suggest possible interventions as it is occurring [Auchincloss and Roux, 2008, Farmer and Foley, 2009]. In ACE, all AB economic models so far have been conceptual and constructed based on hypothetical economies, taking into account some of the characteristics and features of the world economy at a qualitative level, but not matching it in any detail. This lack of realism is due to the challenge of calibrating the AB model to real economy data.

2.6 Summary

The lack of interdisciplinary research in the field of the CDS market has led to the failure of current macroeconomic models to predict the financial crisis of 2007-2010. In particular, it became apparent that it can be misleading and dangerous to rely on a single tool or methodology. In contrast, our re-

search focuses on an interdisciplinary approach to derive statements that are more robust and overcome misjudgements of single scientific fields. This chapter provides a comprehensive review of the four main subject areas that form the basis of this thesis: CDS, AB modelling, data analysis, and CDS pricing techniques.

Section 2.1, gives a comprehensive background of the CDS market. Particularly, the focus is on the primary attributes of the CDS contract, trading, risk, market participants, structure, and data. Our review demonstrates the important role of CDS contract in the 2007-2010 credit crisis and the difficult nature of this derivative contract is discussed. Moreover, the 2007-2010 credit crisis not only revealed the problems of the CDS contract but also the lack of appropriate tools to study these problems and their impact before bringing this product to the market and experiencing its unfortunate effects. In Section 2.2, the AB modelling suitability as an alternative tool to traditional macroeconomic models is projected by presenting its methodology, its theory, and the research conducted so far in the field, describing its practicability for the financial market, and detailing some of the major relevant works in this field. Then it is shown how the traditional methods for calibrating AB models negatively affect the feasibility of AB modelling for real world application in Section 2.3. Section 2.4 discusses the suitability and utility of the traditional CDS pricing techniques for incorporating with AB modelling, and it is shown how important it is to utilise simple and accurate techniques in order to avoid pushing unnecessary complexity to the AB modelling. Section 2.5 illustrates how the ACE field suffers from not calibrating the AB model to real economy data, thus making it inpracticable in the real world.

The next five chapters of this thesis are dedicated to the proposed solution for tackling the challenges highlighted in this chapter. Chapter 3 describes the model formation and the proposed methodology for incorporating empirical data into AB modelling. In Chapter 4, the problem of providing unlimited realistic data for AB modelling is tackled. Chapter 5 explains the suggested regression techniques for pricing a CDS contract. The AB CDS market is introduced in Chapter 6. Chapter 7 illustrates the effect of combining the suggested solutions and the potential application.

Chapter 3

Model Formation

Chapter 1 discussed how the world economy became informed of two crucial issues as a result of the experience of the 2007-2010 financial meltdown: the underestimated impact of some financial products such as Credit Default Swap (CDS) and the serious limitations of current economic and financial models. During the past years, the economic scientific community voiced a serious need for an alternative complementary tool such as Agent Based (AB) modelling to enhance the robustness and practicability of existing models in dealing with real world challenges. Policy makers and regulators can employ such a model to examine the short-term and long-term impacts of financial products (e.g. CDS) on the market, as well as emergent market behaviour. This modelling could result in banning the introduction of risky products such as CDS contracts to the market or issuing a warning in the case of an unusual situation, thus avoiding manageable losses and possible financial crisis. However, as shown in Chapter 2, the current state-of-the-art solutions for using AB modelling to simulate economic phenomena fail in practicality due to two fundamental issues: constructing the AB models based on hypothetical economies and not paying attention to the real world data.

This Chapter presents a unified general framework for the application of AB modelling to the CDS market. The CDS market is chosen for modelling, specifically because of its challenging nature and the crucial role that it played in the recent financial crisis. The approaches presented here are general purpose techniques that can be linked together to advance the Agent-Based Computational Economics (ACE) models. These approaches can be seen as modular building blocks that can be applied in a variety of applications. This thesis presents how the combination of these techniques can prove to be valuable in an ACE application.

The general approach is to enhance the AB models by stochastic processes and regression techniques to leverage the power of the ACE models. The main focus of this research is to tackle the three main challenges associated with simulating the CDS market. This thesis makes contributions in the ACE research field, by proposing novel solutions for the stated challenges. In order to present our contributions in the following chapters, this chapter restates the problem in detail; provides an overview of the approaches employed; proposes a unified model describing our target scenario and its application, as well as stating the assumptions.

3.1 Restating the Problem

The aim of this thesis is to

“investigate the viability of approaches that address the three challenges of lack of sufficient data to support research, lack of efficient CDS pricing technique, and lack of practical CDS market experimental model, that are faced by designers of CDS investigation tools.”

In order to achieve our aim, we identify three main challenges of this work as lack of sufficient data to support research, lack of efficient CDS pricing technique to be integrated into agent based model, and lack of practical CDS Market Experimental Model. Although this research is focused on modelling the CDS market, the proposed solutions are not limited to the CDS market and can be used for any other application which requires a data generative system, a regression tool, or an AB model that uses empirical data.

These three identified challenges are investigated individually, however we will demonstrate how the proposed solutions can be combined to form an AB model which is closer to reality and thus more practical in dealing with real world challenges. Each challenge will be described individually by means of examples from a potential scenario.

3.1.1 Challenge 1: Lack of sufficient data to support research

Section 2.3 discussed how the AB models are traditionally compromised by uniform random distribution due to two critical issues: incapability of AB models for incorporating empirical data due to AB modelling methodology (logic of simulation) and difficulties in obtaining real world data. However, obtaining real data and feeding it into the AB models is not the only challenge. As discussed in 2.3, nowadays, data can be obtained from various sources (see Section 2.2.1). Later in this chapter, it will be explained how the logic of simulation can be modified to tackle the problem of the AB models in dealing with empirical data. Thus the problem of real world data limitation is the first challenge identified in this thesis.

The problem of random initialisation and also limited data length may be illustrated in a scenario where the interest is to study the behaviour of West Airways (refer to the example given at the start of the first chapter) company using AB modelling. In this example, the West Airways company has been launched 5 years ago and the “behaviour” refers to the goodness or badness of the health of the company (e.g. market growth). Assuming that, G is the key indicator of the health of West Airways, X is the indicator of West Airways current financial status (e.g. bond price), and e is an external factor which influences the West Airways company. Next, suppose that the G can be calculated through the following relationship between X and e :

$$G = \frac{X_t - X_{t-1}}{e}$$

where t refers to time ($t = \{1, 2, 3, \dots\}$). Two objectives may be set for this scenario: A and B . Objective A is to study the health of West Airways and investigate the impact of different factors on it. Objective B is to see how the health of West Airways changes over the next 10 years to arrive at the current financial situation.

This investigation may be tackled by using the traditional AB modelling techniques where the AB model is operationalised by a uniform random distribution for initializing the model parameters such as X and e . In this approach, it is possible to simulate a company which has the same functionality and attributes as the West Airways but the model does not share any details regarding the data due to the random distribution. Such a model can be helpful for studying the relationship between the different factors. For instance, it is possible to investigate how strong the X and e factors can affect the G by means of different experiments and tuning the different parameters, thus fulfilling the objective A. However, this model is completely incapable of delivering what object B demands as the current financial situation of the West Airways (the parameters value) is ignored in this approach due to the random initialisation.

To address this problem and achieve the object B, the West Airways real data can be used for initialising some or all of the parameters. This is a valid approach and can provide a model which is more similar to the West Airways company, thus the model behaviour can be more analogous to the real company. However, there are two potential questions to be answered here: is it enough to set the initial value by the empirical data or is there a need to provide the time series? How can the company behaviour over the next 10 years be estimated while only the empirical data for last five years is available? Returning to the example, the initial value of G , X , and e can be set at the beginning of the experiment. The value of these parameters will be derived at time t_1, t_2, \dots, t_i from the different interaction and processes of the model. Therefore, the overall model behaviour will be strongly dependent on the accuracy of the model and the initial abstraction process. Although this approach is a step forward compared to random initialisation, but the model accuracy is doubtful due to the abstraction process.

An alternative approach is to use the time series during the simulation instead of only setting the initial parameters. Using this method the AB model can be fed with the empirical data. For instance, the time series of X and e can be provided for the model. The modeller then can derive the value g using the specified equation over time. This approach is practical for investigating the object A but the object B is not addressable because the simulation ends as soon as the empirical data is finished (5 years in our example). Therefore, future scenarios cannot be studied. So, in addition to the challenges of obtaining the empirical data and injecting it into the AB model, simulating realistic data (data which has the characteristics of real data) for long simulation times (10+ years) is the next challenge to be addressed.

Approach 1: Data Generative Model

Section 2.3.2 discussed why the current approach (random initialisation) is not viable for modelling a real world phenomenon. This Section will introduce a novel solution to tackle the challenge of limited empirical data for AB models called “*Data Acquisition Module*”. The proposed module is outlined as a generative data engine which studies the empirical data, extracts its attributes, and produces similar data for the requested period of time. Figure 3.1.1 is a conceptual illustration of this module and consists of three main processes:

Data Cleaning Process. The raw empirical data, often called dirty data, can be full of errors and, therefore, cannot be used straightaway. These errors, depending on the source, could vary from a

missing value to impossible or inconsistent values or unlikely values. For instance, in a financial market, the value of a bond price is set to “not available” (n/a) when the price is not available due to technical or human error, trade does not occur, or is cancelled. An example of such a data set is

$$X = \{X_t, X_{t+1}, na_{t+2}, X_{t+3}, X_{t+4}, \dots\}$$

where X represent the bond price and na represents the unavailability of data for day t . A data set with such a value (error) is not appropriate for analysis. The task of the data cleaning process is to assure that the data is ready for use in further analyses by checking and correcting all the possible errors.

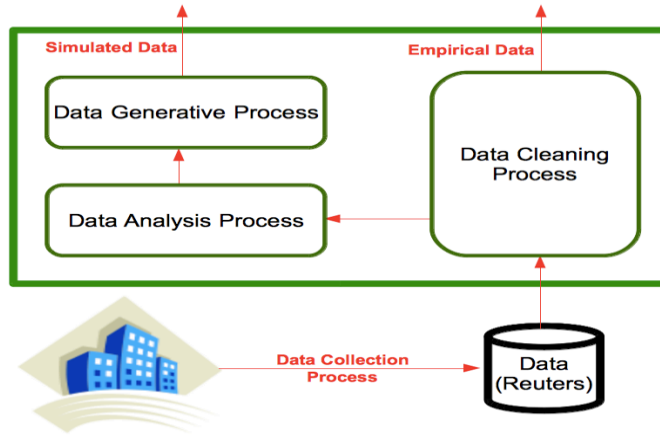


Figure 3.1: Data Acquisition Module

Data Analysis Process. The objective of a data acquisition module is to produce data which has the characteristics of empirical data. Therefore, the task of the data analysis process is to extract these characteristics and features. This task may be explained by means of an example. A data set $X = \{X_t, X_{t+1}, \dots, X_{t+100}\}$ contains the daily bond prices of West Airways from day t to $t + 100$. Based on market information, the West Airways bond price follows a market trend: $Z = \{Z_t, Z_{t+1}, \dots, Z_{t+100}\}$. Now, in order to model the data set X , a knowledge is required of the basic characteristics of the data set X such as its dependency on market trend. It is also important to identify whether the bond price behaviour follows the same pattern or if it follows a different pattern in different time frames. Visual and statistical analysis is used to highlight these characteristics in the database.

Data Generative Process. The final task is to use the extracted features as well as the guessed function to produce a similar data set.

Chapter 4 is devoted to investigating the first challenge, “Data”, where the details of this approach and techniques are provided. It will also be shown why the chosen technique is promising through a critical literature review. The contributions are also presented and discussed as appropriate.

The results of a comparative evaluation against the benchmark are reported and the efficiency of the proposed model is discussed.

3.1.2 Challenge 2: Lack of efficient CDS pricing technique

Section 2.4 demonstrated how the CDS contract can be priced using two grounded theories: the Probability Model and the No-arbitrage Model. Consider the behaviour of two traders who are getting involved in a CDS contract. An AB model consists of two types of agent: trader agents and a risky company agent. A trader agent represents the institutes (such as North Bank or East Bank - referring to the example given at the start of the first chapter) who buy or sell CDS contracts. The risky company agent represents West Airways. Each trader requires a pricing method to evaluate the price of the CDS contract based on the available information of the risky company (e.g. West Airways).



Figure 3.2: CDS Pricing Negotiation

The accuracy of the pricing method is important because it affects the traders financial situation as well as the overall model behaviour. For example, under pricing a CDS contract based on West Airways can lead to a general market belief that the West Airways is in a healthy condition as the CDS contract is an indicator of the health of a company. This belief might lead to pushing up the West Airways stock price, consequently making fortunes for the stock holders and an unpredicted loss for CDS sellers.

For instance, assume that North Bank and East Bank become involved in a one year CDS contract. If the West Airways defaults, East Bank has to pay the compensating payment based on the agreed recovery rate to North Bank. The recovery rate is one of the CDS contract terms which is agreed by both dealers. By selling a CDS contract, East Bank accepts the risk of a West Airways default. However, the true pricing of the contract enables East Bank to have a better understanding of West Airways condition of health, thus setting a lower recovery rate, or selling the contract with a higher price to manage the losses if the default risk is high. Similarly, North Bank can bargain on the contract if the price indicates that the West Airways condition of health is fine and the risk of default is low.

In Section 2.4, it was argued how the Probability Model is insufficient for cooperating with AB models due to the level of complexity that enforce to the model and how the No-arbitrage Model is inaccurate due to its existing arbitrage channel. Therefore, a modeller would seek a pricing technique which is capable of valuing the CDS contract accurately without enforcing the extra complexity of the AB model.

Approach 2: CDS Pricing Center

This Section will introduce a novel approach to tackle the challenge of CDS pricing for AB models called “*CDS Pricing Module*”. The proposed module is outlined as a regression engine which studies the empirical data (inputs and output) and tries to find the best relationship between the input variables which provides the best output. The main goal of this module is to provide a pricing tool for trader agents in the AB model.

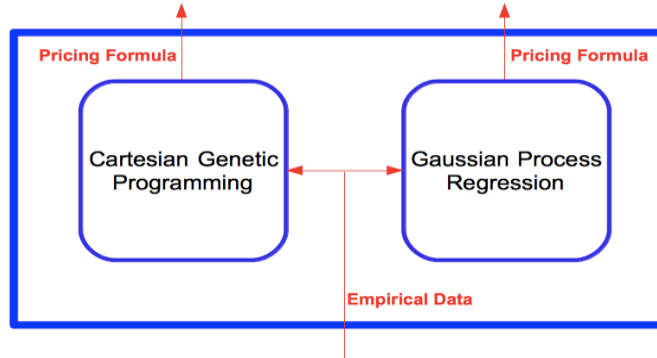


Figure 3.3: CDS Pricing Module

Figure 3.3 is a conceptual illustration of this module consisting of two different types of technique: a statistical technique called Gaussian Process Regression (GPR) and an evolutionary technique called Cartesian Genetic Programming (CGP). These two techniques are two of the most commonly used regression techniques and have been applied successfully in different fields, as will be discussed in 5, but have not been employed for the purpose of catering the AB models to date. Chapter 5 is devoted to investigating our second challenge, “*CDS Pricing Technique*”, where we provide the details of our approach, techniques, and contributions.

The CDS pricing approach will be evaluated by means of comparing the obtained result against the theoretical benchmark.

3.1.3 Challenge 3: Lack of practical CDS market experimental model

Chapter 1 described how the CDS contract is widely blamed for its key impact on the 2007-2010 financial crisis as well as discussing the failure of traditional microeconomic models in helping regulators and policy makers during the financial crisis. This failure could be the result of two basic behaviours by economists: lack of attention to the underlying fast individual movements, and over trusting and relying on the conventional and traditional tools for many years.

Looking at the world economy on two different levels there is a financial institution level and a regulator level. In the last decade, as usual, financial institutions such as banks or hedge funds were trying to employ the brightest people and rewarding them generously for taking advantage of the newest technology, for developing new techniques and improving the existing tools and strategies, to enhance their performance, consequently increasing their profit. From a financial institution level, the highest priority in the business is to enhance the profit. Therefore, there is no surprise when the public preference

has no importance as each individual cares for him or herself by employing best of all, keeping secret the profitable information or techniques, and mis-reporting sometimes. In contrast, what should matter from a regulators point of view is supposed to be the public preference and profit.

Unlike a financial institution which regularly tries to update its tools and techniques to increase its profit, economists believed that they were well secured with what they had in their hands, traditional microeconomic models, to understand what was happening in the world economy. However, the recent financial crisis exhibited the limitations of the traditional microeconomic models. Mr Trichet expressed and disclosed these limitations by calling for developing complementary tools to improve the robustness of the overall framework in 2010 [Trichet, 2010]. An example of this limitation is the incapability of existing microeconomic models to identify the CDS contracts problems, thus failing to avoid the CDS market development at the crucial time leading to fortunes being made and lost during the recent financial crisis.

This thesis is focused on the CDS market and it is the first attempt at simulating the CDS market in order to deliver a feasible experimental tool in which the different market participants, elements, behaviours, and events can be studied under the various laboratory conditions and situations. By successfully achieving this aim, it will be illustrated how the different techniques and complementary tools can cooperate with each other in order to form a practical experimental tool.

Approach 3: CDS market Experimental Model.

Chapter 2 explained how 2010 became the year that the potential of AB modelling became recognised by the economic community [Anon, 2010a] after the market crashed because of current traditional microeconomic techniques. However, as derived by Section 2.5 of the background Chapter, it has been some time that AB models have been used for studying the market components and behaviour, but the practicability and feasibility of this model in dealing with real world problems has been downgraded due to the range of concerns and challenges. This section introduces a novel solution for modelling the CDS market using AB modelling. A solution will be outlined by providing an overview of approaches for modelling different components of the AB model.

A CDS market consists of four types of key players: a trader, a risky company, a credit rating agency and a regulator. Traders are those who buy and sell CDS contracts. A risky company is a reference entity of a CDS contract. A credit rating agency is an independent organisation which regularly monitors the market components and publishes their financial situation. A regulator, such as a government or a Financial Service Authority (FSA), is responsible for setting the general factors such as an interest rate and policies, and monitoring the players activities. Figure 3.4 illustrates a conceptual definition of this CDS agent based model. As presented, there are two types of negotiation in the market: negotiations between the traders (CDS deal) and negotiations between a trader and a risky company (loan deal).

Chapter 6 is devoted to investigating a third challenge, “*CDS Market Experimental Model*”, where the details of our agent based CDS market are provided and its components, including the players characteristics and interactions as well as the proposed techniques for modelling each of the components, and presenting and discussing this thisss contributions. The proposed model is discussed by comparing

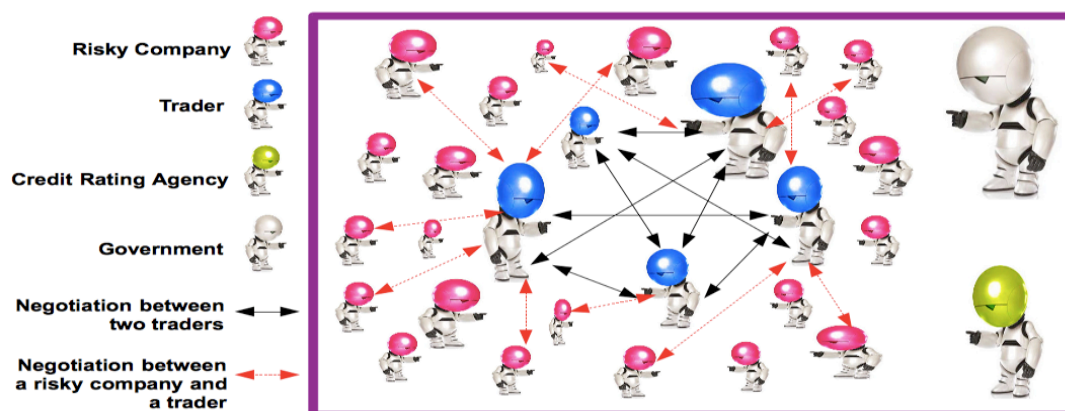


Figure 3.4: Agent Based Model

the result with actual human behaviour.

3.2 CDS Agent-Based Virtual Economy

Section 3.1.1, 3.1.2, and 3.1.3 reviewed the proposed approaches to tackle the challenges of data, CDS pricing, and CDS modelling. In the following chapters, there is a discussion of how each of these individual approaches can contribute to the ACE field and prove to be beneficial for investigating the CDS market. However, in this section it is argued that relying on specific techniques and models can restrict insight. If the real data were to cover the period of three years and within these three years the CDS market has been very stable. By using the AB CDS market and real world data, it will be possible to study the traders behaviour in the stable market, however, it will not be possible to provide any assumptions for an unstable market.

The final challenge of this thesis is to illustrate how the different techniques and complementary tools can cooperate with each other in order to form a practical experimental tool for studying the CDS market and prove the proposed models efficiency in dealing with real world problems. It is proposed to integrate the AB model, data acquisition module and pricing module to form a comprehensive experimental tool called CDS agent based virtual economy. By doing so the practicability and capabilities of the AB CDS market is enhanced.

It is illustrated how the combination of these general purpose approaches can improve the overall findings by studying the CDS market using different scenarios and answering specific scientific questions. Chapter 7 explains the concerns and objective as well as discussing the experimental result. The result is validated by discussing how these findings relate to real world events.

Chapter 4

Data Analysis

As discussed in Section 2.3, the Agent Based (AB) models are traditionally catered by uniform random distribution because of two critical issues: the incapability of AB models to incorporate empirical data due to the AB modelling methodology (logic of simulation) and difficulties in obtaining real world data. Section 3.1.1 discussed the negative sides of using the random initialisation techniques to cater the AB models. The importance of collecting real data and collaborating it in the AB modelling was also explained using an example where the series of sources (see Section 2.2.1) that provide the real world data were listed. However, different industrial sectors have different security sensitivity and therefore there are several challenges for the data collection process. Although the availability of the real data is not the centre of attention, the limited length of real word data is an important challenge within the Agent-Based Computational Economics (ACE) community.

Our real world data covers only a relatively small amount of time (5 years). However, in Section 3.1.1 it was pointed out how important it is to be able to produce artificial data which shares similarity with real world data in order to simulate the market for longer periods, such as 10, 15, or 20 years. The earlier example in Section 3.1.1 explains how the real world data limitation restricts the investigation of future scenarios, thus affecting the AB models practicability for real world application.

In this chapter, the first challenge is addressed, “lack of sufficient data to support research” so the problem of limited empirical data for catering ACE models is tackled. A novel solution is introduced with an objective of providing a generative data engine which studies the empirical data, extracts its attributes, and produces data with similar attributes for an specified period of time. The solution is implemented as a generic modular approach so that it may be reused by other systems in the future.

As explained in Section 3.1.1, the proposed module consists of three processes: the *Data Cleaning Process*, the *Data Analysis Process*, and the *Data Generative Process*. Section 4.1 presents our empirical data and explains our data cleaning process. The data analysis process is described in Section 4.2 where data key features are discussed. Section 4.3 introduces the data generative process where the state-of-the-art solution is described along with the motivation for the approach, and implementation details. The results are also discussed in this Section. Section 4.4 validates the result. This validation is followed by a summary of the findings as well as conclusions in Section 4.5.

4.1 Data Cleaning Process

Twenty companies from iTraxx¹ energy market have been chosen for these experiments. This dataset is chosen because of its accuracy and availability by the time that this research was in progress. Data was collected² from 1st January 2004 till 25th June 2009 (which includes the recent highly turbulent period of the markets).

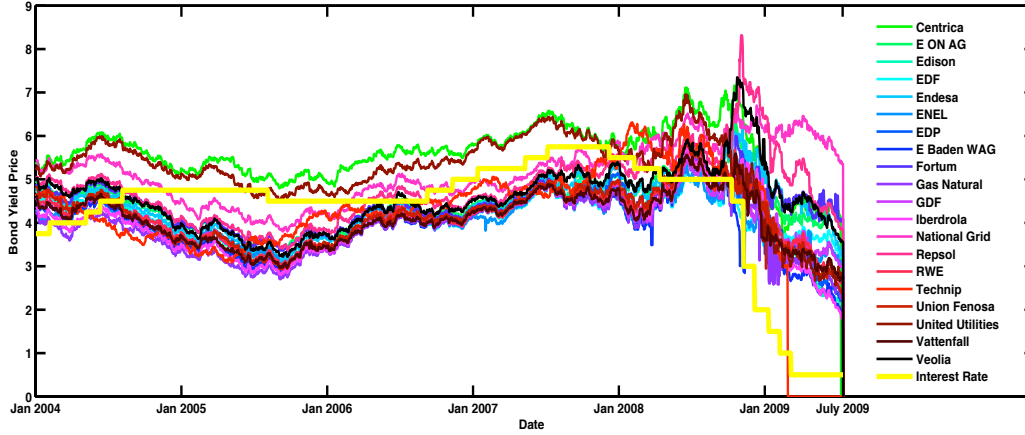


Figure 4.1: Real-world iTraxx Energy Data

Figure 4.1 illustrates the bond prices of these twenty companies as well as the interest rates for the mentioned period. The bond is the debt indicator of the company and the interest rate is a global factor which has impact on business performance. Generally speaking both the bond and the interest rate can be used to assess the performance and the default risk of a company.

As explained in Section 3.1.1, the raw empirical data might have errors and cannot be used straight away. Table 4.1 demonstrates a sample of the raw data. Naturally, the financial data are irregularly spaced in time. This irregularity is due to a lack of transactions on some days or the exchange closure during the weekend and public holidays. These data are called an inhomogeneous time series. However, most of the time series analysis methods are based on a homogeneous time series, where data are regularly spaced in time [Schmid, 2009].

Our raw data is inhomogeneous due to the missing information. This missing information is stated as “#N/A” in Table 4.1 where YLD_{BID} stands for bid³ price of bond yield, YLD_{ASK} stands for ask⁴ price of bond yield, and CDS_{spread} stands for CDS price.

To tackle this problem a popular technique is used which is called the *interpolation method*. Generally speaking, interpolation is a method for creating a new artificial data point between two existing

¹iTraxx is the brand name for the family of credit default swap index products covering regions of Europe, Australia, Japan and non-Japan Asia. They form a large sector of the overall credit derivative market.

²These historical data are available for collection from Reuters (www.reuters.com) and Bloomberg (www.bloomberg.com).

³A bid price is the highest price that a buyer is willing to pay for a good.

⁴An ask price is the lowest price that a seller is willing to accept for a good.

Table 4.1: A Snapshot of Centrica Company Data.

Date	YLD_{BID}	YLD_{ASK}	CDS_{Spread}
27/02/2007	5.813	5.773	19.85
28/02/2007	5.809	5.77	20.41
01/03/2007	5.807	5.767	#N/A
02/03/2007	5.789	5.748	19.44

data points. There are two important interpolation methods: linear interpolation and previous-tick interpolation. In practice, the difference given by the two different interpolation methods is negligible [Dacorogna et al., 2001, Schmid, 2009]. However, this claim might not hold for all datasets. For instance, if there were 50 missing items in a row, then the differences could become more significant depending on the data fluctuations. Linear interpolation was chosen because it is a well-established and proven approach for the field of financial computing when there are few missing data points and data is evenly distributed [Wiener, 1949, Kokic, 2002]. The linear interpolation for datapoint i is defined as

$$X_i = \frac{X_{i+1} + X_{i-1}}{2}$$

while the previous-tick interpolation for the same datapoint is defined as

$$X_i = X_{i-1}$$

Figure 4.2 illustrates the result of two interpolation methods for the missing CDS_{spread} on date 01/03/2007 in Table 4.1. In this thesis the previous-tick interpolation method is used to deal with missing data. After filtering and transforming the data to clean the data then we can perform our analysis process to extract data features and characteristics.

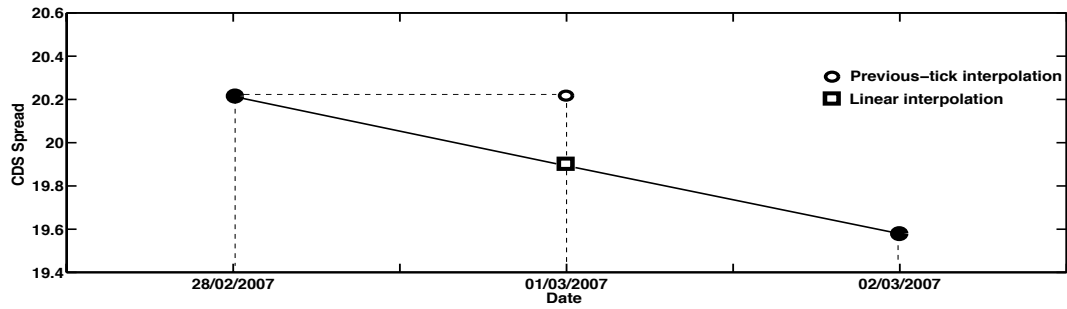


Figure 4.2: Different Interpolation Methods. We use the previous-tick interpolation method in this thesis, therefore, the missing datum for 01/03/2007 is filled by 20.2 which is the CDS price for 28/02/2007.

4.2 Data Analysis Process

The process of analysis is begun by visualising the data. Figure 4.1 illustrates the bond prices of twenty companies. The figure suggests two points:

1. There seems to be a common trend that all the companies follow to some extent.

2. The behaviour of bond prices from January 2004 to January 2008 is different compared to their behaviour from January 2008 onward.

Both points reflect the dependency of companies on global effects. It is well known that factors like interest rates, the oil price, political situations or wars can have a strong impact on the economy and on companies. It is furthermore assumed that global factors influence companies from the same industry sector in a similar way. As all our companies are from the energy market it is to be expected that their bond prices change similarly, based on such global effects. These global effects are summarised in a variable which is called the *market trend*.

For simplicity the market trend is modelled as the arithmetic mean of the company bond prices and the market trend is denoted with m . The market trend at time t is $m_t = \frac{1}{n} \sum_{i=1}^n X_t^{(i)}$, where $n = 20$ is the number of companies and i is the index of the company. Obviously, some companies will be more strongly related to the market trend than others.

In the next step how strongly a company follows the market trend is identified. For this, the Pearson correlation coefficient is used which is a statistical tool to measure the strength of linear dependency between two variables X and Y [Rodgers and Nicewander, 1988]. In other words, a coefficient of correlation measures how much one quantity (such as a bond price) can be expected to be influenced by changes in another (such as market trend). The correlation coefficient is mathematically defined as ρ , where

$$\rho = \frac{\sum_{t=1}^T (X_t - \bar{X})(Y_t - \bar{Y})}{\sqrt{\sum_{t=1}^T (X_t - \bar{X})^2} \sqrt{\sum_{t=1}^T (Y_t - \bar{Y})^2}}, \text{ and } \bar{Y} = \sum_{t=1}^T Y_t, \bar{X} = \sum_{t=1}^T X_t.$$

Table 4.2 reports the correlation coefficient between the twenty companies and the calculated market trend. For instance, company number 1s behaviour is 0.7 correlated to the market trend. Some companies show a strong dependence on the market trend (e.g. company no. 7, 15, and 20) while others (e.g. company no 8, 16, and 19) are only weakly correlated.

Table 4.2: Companies Correlation Coefficient.

Company	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Correlation	0.70	0.79	0.78	0.70	0.73	0.55	0.81	0.21	0.63	0.45	0.73	0.78	0.78	0.65	0.90	0.14	0.56	0.51	0.19	0.80

Looking at our data something else is apparent: The bond price distribution of these companies changes at some time in 2007. Through a financial news survey, it became apparent that the financial crisis of 2007- 2008 played a substantial role in the failure of many businesses, and the literature dates the start of this period of turbulence to around 7th August 2007⁵.

We wanted to quantify this effect with an objective measure of the distribution of our data to confirm that the process characteristics of the bond prices changed in August 2007. The classical way to do so is to measure the variance in the bond prices per company. A high variance indicates that the bond prices are fluctuating significantly while a low variance indicates that the bond price is very stable. Table 4.3

⁵For further information, see “Three myths that sustain the economic crisis” by Larry Elliott, economics editor of The Guardian at <http://www.guardian.co.uk/business/economics-blog/2012/aug/05/economic-crisis-myths-sustain>.

shows the variance in the bond prices of our companies, where the variance is defined as $\mathbb{V}(X)$ with

$$\mathbb{V}(X) = \frac{1}{T} \sum_{t=1}^T (X_t - \bar{X})^2.$$

Table 4.3 presents the variance of the companies in three forms: measured over the *full dataset*, over the *normal time* until August 2007, and *crisis time* from August 2007 onwards. Of interest is to check whether the distribution of data is different between the normal and crisis times. As Table 4.3 reports, the data variance in the crisis time is on average 3.464 times higher than in the normal time. Hence, there is significant difference in the bond price characteristics before and after 2007.

Table 4.3: Variance of Bond Prices.

Company	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Full Dataset	0.39	0.31	0.49	0.21	0.37	0.26	0.46	0.46	0.24	0.49	0.29	0.51	0.47	0.60	0.35	1.66	0.43	0.67	0.36	0.51
Normal Time	0.10	0.15	0.24	0.17	0.18	0.14	0.20	0.21	0.16	0.21	0.19	0.20	0.20	0.20	0.18	0.24	0.22	0.15	0.18	0.22
Crisis Time	0.77	0.24	0.67	0.19	0.20	0.34	0.56	0.73	0.14	0.62	0.28	0.73	0.15	0.45	0.41	3.74	0.53	1.45	0.45	0.41

This difference has implications this our data generative model. The aim is to derive models that simulate bond price behaviour of real companies but this data set covers two very different bond price behaviours. It is, therefore, reasonable to split this data into two parts, the part before August 2007 and the part starting August 2007 and to infer two models, one for normal times and one for financial crisis times.

At the beginning of this section the market trend was defined and also the correlation coefficient, ρ , of each company to the market trend was computed. However, after observing the difference in the data distribution and dividing the data into the category of normal time and crisis time, we performed our correlation coefficient analysis on each category in order to investigate the dependency of each company on the market trend in each category. Table 4.4 presents our companies dependency where the first row represents the 20 companies and the first column represents the datasets: normal time and crisis time. For instance, company number 1 has the correlation coefficient of 0.82 (column 2, row 2) in the normal time, while it has a correlation coefficient of 0.66 in the crisis time. The dependency of companies on the market trend decreases by 25% on average in the crisis time, reflecting a higher variation between the company bond prices and a lower stability of the market.

Table 4.4: Companies Correlation Coefficient in Normal Time and Crisis Time.

Company	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Normal time	0.82	0.97	0.97	0.97	0.97	0.90	0.97	0.16	0.97	0.95	0.98	0.97	0.95	0.96	0.98	0.08	0.96	0.80	0.16	0.97
Crisis time	0.66	0.73	0.71	0.62	0.67	0.49	0.76	0.23	0.57	0.41	0.67	0.72	0.73	0.59	0.87	0.16	0.49	0.46	0.20	0.75

After the data analysis we are now ready to tackle our first challenge: a lack of real world data. Section 4.3 introduces a data generative process to address this problem.

4.3 Data Generative Process

In this section, we introduce a data generative model that simulates the real world data thus enabling open-ended simulations driven by an unlimited amount of realistic market data. This model consist of

two generative models: the first model, which we call the *independent model*, generates data independently for the different entities like the different companies, the interest rate and the market trend. The second model, which is called the *dependent model* is an advanced version of the independent model where the behaviour of the companies is correlated with a common market factor which is the market trend in this case. For the dependent model the market trend is used as defined in Section 4.2 and the companies are coupled to the market trend in dependence of how high is the correlation coefficient ρ between the company and the market trend in Table 4.4.

Integrating the market trend and the correlation coefficient factor is crucial for this model as one industry sector is modelled and, therefore, the companies show a strong similarity to bond price behaviour in the real world which is to be simulated.

To achieve this goal, a well established random walk technique is used called *Lévy process* [Applebaum, 2004] and it is adapted to the data with the help of the *maximum likelihood* technique. By using these methods an assumption is made that the many factors in the real world that affect market trends can be simulated through random models. This is seen as a best compromise as it is not feasible to model all the factors (which may be highly unpredictable because of factors such as politics or fashion) that affect the markets. Generally speaking, the maximum likelihood technique is utilised to extract key features of the real world data. The Lévy process is then adapted to these key features to generate data simulating key properties of the real world data. This adaptation enables the simulation of realistic data sets as well as providing the leverage of simulating different realisations of potential bond price behaviour per company. I.e. a Lévy process is a stochastic process and two simulations of a Lévy process are, essentially, never identical.

In the next section the motivation for utilizing Lévy process is explained. The details of our independent and dependent model are given in Sections 4.3.3 and 4.3.4. The results are discussed in Section 4.3.5.

4.3.1 Motivation

Stochastic processes like the Lévy process are well established in many branches of mathematics, engineering and in economical science to represent and model stochastic systems. For a long time the dominant stochastic process technique was *Brownian motion*.

The Brownian motion phenomenon came to be known as a result of the experiments of Robert Brown, a Scottish biologist. Brown discovered the random behaviour of pollen particles suspended in water in 1827 [Brown, 1828]. 80 years later, Albert Einstein developed the mathematical properties of Brownian motion and presented it as an indirect way to confirm the existence of atoms and molecules [Einstein, 1905]. At the same time, around 1900, Louis Bachelier observed the *random walk* behaviour in financial markets and proposed that they follow a random walk which can be modelled by standard probability calculus [Bachelier, 1900]. A random walk is, basically, a Brownian motion where the previous change in the value of a variable is unrelated to future or past changes. Since then, Brownian motion became a central concept in stochastic calculus which is used to model systems that behave randomly

with countless numbers of applications⁶ in different fields of medical imaging, robotics, estimation of extreme floods and droughts, manufacturing, decision making, aerosol particles, aerosol transport phenomena, particle disposition on the human nose and mouth, and laser evaporation of copper aerosol.

In the financial world, Brownian motion is an attractive technique due to its desirable mathematical characteristics, where statistics can be estimated with great precision, and probabilities can be calculated. Thus scientists and analysts utilise the method to model processes of unknown origin such as the stock market. The Brownian motion theory and random walk model are widely applied to the modelling of markets since Bachelier's first work and this continues to this day. However, there are two main concerns about the Brownian motion within the financial community: sample paths from a Brownian motion are continuous and infinitesimal increments are Gaussian [Papapantoleon, 2008]. In the real world, it can be observed that the financial prices such as bond prices have jumps or spikes, see Figure 4.1, which are discontinuous and non-Gaussian. These jumps (big price changes) are very common in financial markets. The Brownian motion is incapable of producing these jumps. Moreover, the empirical distribution of asset returns exhibits heavy tails and skewness, behaviour that deviates from Gaussian increments and thus from the characteristics of Brownian motion.

A Lévy process, named after the French mathematician Paul Lévy, is a stochastic process with independent, stationary increments: it represents the motion of a point whose successive displacements are random and independent, and statistically identical over different time intervals of the same length. A Lévy process may thus be viewed as the continuous-time analogue of a random walk. Lévy processes are becoming extremely fashionable in mathematical finance due to their capability in describing the observed reality of financial markets in a more accurate way than models based on Brownian motion, thus providing more appropriate tools to adequately and consistently describe all these observations such as price jumps, heavy tails and skewness [Papapantoleon, 2008]. Lévy processes play a central role in several fields of science, such as physics, in the study of turbulence, laser cooling and in quantum field theory; in engineering, for the study of networks, queues and dams; in economics, for continuous time-series models; in actuarial science, for the calculation of insurance and re-insurance risk; and, of course, in mathematical finance [Papapantoleon, 2008]. A comprehensive overview of several applications of Lévy processes can be found in Prabhu [Prabhu, 1998], in Barndorff-Nielsen, Mikosch, and Resnick [Barndorff-Nielsen et al., 2001], and in Kyprianou [Kyprianou, 2006].

4.3.2 Mathematical Definition of a Lévy Process

A Lévy process is a continuous time stochastic process. Formally, a stochastic process consists of a family of random variables indexed by time t . An observation from a stochastic process is a sample path $(X_t, t \geq 0)$, which could, for example, be used to represent bond prices evolving over time. A Lévy process is now a stochastic process that has the following three properties [Applebaum, 2004][Sec. 1.3]:

1. $X_0 = 0$ with probability 1.

⁶See the report on Brownian Motion, The Research Goes On... by Y.K.Lee and Kelvin Hoon at http://www.doc.ic.ac.uk/~nd/surprise_95/journal/vol4/ykl/report.html.

2. X has stationary increments. Stationary means that the distribution of the increments of the process is at all times the same.
3. X has independent increments. Independent increments means that an increment from time s to t of the process is independent of any increments that occurred before time s .

While Lévy processes are continuous time processes, they are also very well suited to describe objects like bond prices. The approach here is that it can be assumed that the bond price is actually a continuous curve which can be observed only at discrete times. One advantage of this approach is that the time when the observations occur does not matter. In contrast, many discrete time methods rely on equally spaced observations.

4.3.3 Independent Model

The independent model in this research is built around Lévy processes to simulate, for example, Bond prices from companies. The Lévy process is used here as a generative model, that is a model which can generate data. There are a number of free variables in our Lévy process which can be used to adapt the generative model to the data at hand. Here a maximum likelihood approach is used to infer parameters that make the process produce paths which are similar to the real world data.

Particular Lévy processes are considered which consists of the sum of a Brownian motion and a compound Poisson process. The Brownian motion part can model small scale randomness in, for example, bond prices. The Poisson part, on the other hand, is useful to model major events that lead to bigger jumps in the bond price.

The compound Poisson part of the Lévy process is a combination of a Poisson process which simulates waiting times, i.e. it models, for example, the time that passes between major financial events and a distribution for the jump height. For this an univariate Gaussian distribution is used. There are three parameters of the Lévy process which are :

- σ_B is the scale of the Brownian motion and describes the small fluctuations,
- λ is the jump rate of the compound Poisson process, i.e. it describes how often bigger jumps occur in the process, and,
- σ_H is the scale for the jumps of compound Poisson process part, i.e. the standard deviation of the Gaussian distribution.

In the next section an alternative and intuitive interpretation of the model is given. The following two sections contain more details about the Lévy process which is used and about the maximum likelihood inference.

Interpretation as a Hidden-Markov Model

This model has an intuitive description as a two state Hidden-Markov Model (HMM) [Rabiner, 1989]. HMMs are a well established tool in computer science to represent stochastic systems. Figure 4.3 illustrates the HMM model.

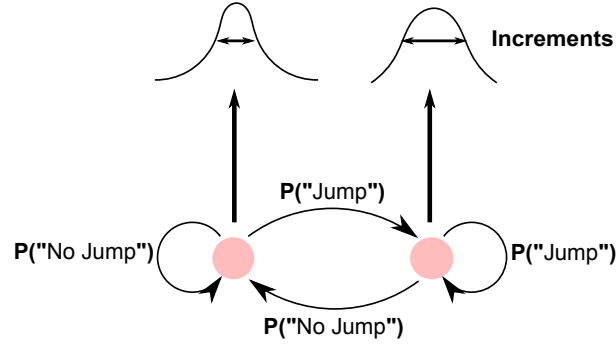


Figure 4.3: Conceptual Visualization of the Independent Model

The HMM has two states, represented by the red circles, and the system is at each time step t in exactly one of these states. After each step the system transits to a new state with a certain probability, which is written beside the corresponding arrow. At each time step the system emits a signal which in this case represents the increment in the company bond price. The left state represents the case that no jump occurs at a given time and the increments are completely described by the Brownian motion part. This is a small increment compared to the bigger jumps. It is visualized by drawing a Gaussian distribution with a small standard deviation on top of the left state. The right state represents a jump and consequently it has a Gaussian distribution with a bigger standard deviation. The system starts in the left state, i.e. the no jump state. It is worth noticing that the probability for a jump is independent of the state the system is currently in as both arrows leading to the right state have the same probability $P(\text{"Jump"})$.

The company bond prices are now simply the sum of the increments that are observed from the HMM model.

Details of the parametric model

For practical purposes it is useful to know that a Brownian motion is a Gaussian process with mean 0 and with covariance $\min(t_i, t_{i+1})$ between times t_i and t_{i+1} . The importance of this observation lies in the fact that it is easy to simulate data from a Gaussian process and to perform inference.

In this a model a scaling factor of σ_B is used to make the Brownian motion adaptable to the data at hand, i.e. a Gaussian process with mean 0 and covariance

$$\sigma_B \min(t_i, t_{i+1}). \quad (4.1)$$

In [Williams and Rasmussen, 2005] a generic way is described to sample from Gaussian process. The following algorithm is a version of this procedure adapted to this specific Gaussian process. Figure 4.4(a) shows a sample from the scaled Brownian motion which is generated with Algorithm 1.

The compound Poisson process generates jumps at random times, where the jump height is stochastic based on a distribution. The random times are drawn from a Poisson process with rate λ and a Gaussian distribution is used for the jump height: $\mathcal{N}(0, \sigma_H)$. Figure 4.4(b) shows a sample from this process.

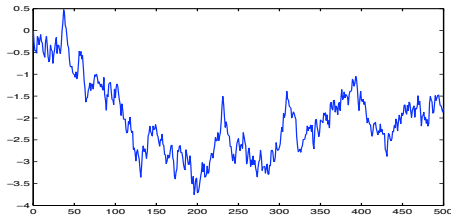
To get samples for the Lévy process first a path $f(t)$ is generated from the scaled Brownian motion

Algorithm 1 Sampler for Brownian Motion – Pseudo Code (Matlab)

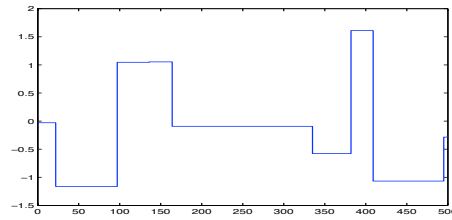
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1: Input  $Bt$ : A vector of times  $0 \leq t_1 \leq \dots \leq t_n$  at which sample values should be produced.
2: Output  $BrownianSamplePath$ : The sample of length  $n$  from our scaled Brownian motion.
3: for  $j = 1$  To  $length(Bt)$  do
4:    $j$  goes through all  $n$  indexes for the times  $t_1, \dots, t_n$ 
5:   for  $i = 1$  To  $length(Bt)$  do
6:     % same for  $i$ 
7:      $K(j, i) = \sigma \min(Bt(i), Bt(j))$  %  $\min(x, y)$  is the minimum of  $x$  and  $y$ . This line calculates
8:     % the covariance of the process at times  $t_i$  and  $t_j$  and stores it in the matrix  $K(j, i)$ .
9:   end for
10: end for
11:  $L = chol(K + 0.00001 * eye(size(K))), 'lower'$ 
12: % This is a standard approach to sample from a Gaussian process.  $chol$  with the parameter 'lower' is a Matlab function that
    calculates a Cholesky decomposition of the matrix  $K$  into a lower triangular matrix  $L$  such that  $K = LL'$ , where  $'$  is the
    transpose. The  $eye$  command generates a diagonal matrix of the same size as  $K$  with ones on the diagonal. The corresponding
    term is added to  $K$  to make the matrix better conditioned such that the Cholesky decomposition algorithm does not run into
    numerical problems.
13:  $u = normrnd(zeros(size(Bt)), 1)$  %  $normrnd$  is a Matlab command to produce samples from a Normal distribution. This
    command is called to produce a  $n$ -dimensional vector of values drawn from a Normal distribution with standard deviation 1
    and mean 0.
14:  $BrownianDataset = L * u'$ 

```



(a) Brownian Motion



(b) Compound Poisson process

Figure 4.4: Decomposition of Levy Process

and then a path $g(t)$ from the compound Poisson process. The path $h(t)$ from the generating model is then simply $h(t) = f(t) + g(t)$. Figure 4.5 shows a sample from this process. One can clearly observe the small scale variations from the Brownian motion and the larger jumps from the compound Poisson process.

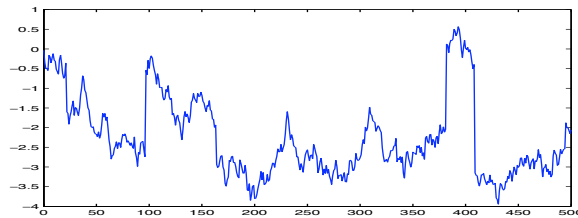


Figure 4.5: Levy Process

Algorithm 2 Sampler for Compound Poisson – Pseudo Code (Matlab)

```

1: Input  $\lambda$ ,  $T_{max}$ , and  $\sigma_H$ : The jump rate  $\lambda$  that determines how a man jumps per unit of time we expect, the time  $T_{max}$  until
   which the jumps are sampled and the parameter  $\sigma_H$  for the jump height.
2: Output EventTime, JumpsHeight: The list of jump times and the corresponding jump height.
3:  $EventTime(1) = random('Exponential', 1/\lambda)$  % Set the time of the first jump. This is the same as the amount of time
   needed to wait for the first jump. This waiting time is exponentially distributed/ The Matlab command random with parameter
   'Exponential' produces a sample from such an exponential distribution with parameter  $1/\lambda$ .
4:  $m = 1$  % Draw further events until time is up
5: while EventTime(m) <  $T_{max}$  do
6:    $EventTime(m+1) = EventTime(m) + random('Exponential', 1/\lambda)$  % The time of the next jump is calculated
   as the time of the last jump plus a new waiting time which is obtained from the exponential distribution.
7:    $m = m + 1$ 
8: end while
9:  $JumpsHeight = normrnd(zeros(m, 1), \sigma_H)$  % Draw for each event a jump height from a Normal distribution.

```

Parameter inference

The maximum likelihood method is used [Applebaum, 2004] to adjust the three parameters σ_B , λ and σ_H of the model to represent the real world data in the best possible way. Maximum likelihood selects the parameters which, as the name suggests, maximises the likelihood or probability of the data given to the model. For example, if only rarely jumps are observed, but jumps of high magnitude, then a good model for this will be a model with a low rate of jumps λ and a high average jump height σ_H .

Data likelihood for given parameters. The first thing needed to calculate is the likelihood that is to be optimized. Without this maximum likelihood is not applicable.

Our Lévy process model has the three parameters σ_B , λ and σ_H and a calculation of the probability is required to observe a sequence $(X_t, t \geq 0)$ given these parameters, i.e. calculate calculation of $\mathbb{P}[(X_t, t \geq 0) | \sigma_B, \lambda, \sigma_H]$.

The sequence $(X_t)_{t \geq 0}$ is in this case discrete with observation times t_1, t_2, \dots, t_n and equal spacing $t_{i+1} - t_i = \Delta t$ independent of the index i .

An important property of a Lévy process is that it is a Markov process [Applebaum, 2004][Ex 3.1.1]. This means that the value generated at a time step t_{i+1} depends only on the value t_i and not on the values before t_i . This allows the likelihood to be calculated as a product of likelihoods for single steps (suppressing the parameter dependence for readability):

$$\mathbb{P}[(X_t, t \geq 0)] = \mathbb{P}[X_0] \mathbb{P}[X_1 | X_0] \cdot \dots \cdot \mathbb{P}[X_n | X_{n-1}]. \quad (4.2)$$

Therefore, the probability, or likelihood of an observed sequence is completely defined by the probabilities $\mathbb{P}[X_{i+1} | X_i]$.

This probability depends on how many jumps occur in the time interval Δt . This probability is approximated by assuming that either one jump or no jump occurs and the probability for multiple jumps is neglected, which is very small.

The probability of k jumps in the interval Δt is Poisson distributed with parameter $\lambda \Delta t$. For a

single jump this is equal to

$$\exp(\lambda\Delta t)\lambda\Delta t.$$

Using a probability of

$$1 - \exp(\lambda\Delta t)\lambda\Delta t$$

as the probability for no jump which deviates slightly from the Poisson distribution since the possibilities of more than one jump are neglected.

If no jump occurs then any fluctuation is based on the Brownian motion part of the model and the probability to observe a change of $\Delta X_i = X_{i+1} - X_i$ is given by Gaussian distribution with a mean 0 and standard deviation $\sigma_B\Delta t$ using Equation 4.1 with $\sigma_B \min(\Delta t, \Delta t)$. Writing down the Gaussian distribution the probability is obtained as

$$\frac{1}{\sqrt{2\pi\sigma_B\Delta t}} \exp\left(-\frac{\Delta X_i^2}{2\sigma_B\Delta t}\right).$$

If one jump occurs, then the probability of observing a change of Δx_i is given by the sum of the Gaussians, i.e. the effect of the small fluctuations modelled with the Brownian motion and the effect of the bigger jumps modelled with a univariate Gaussian add up. The sum of two Gaussian variables is again Gaussian distributed with the sum of the means, which is zero, and the sum of the standard deviations which are $\sigma_B\Delta t$ and σ_H . The corresponding probability is

$$\frac{1}{\sqrt{2\pi(\sigma_B\Delta t + \sigma_H)}} \exp\left(-\frac{\Delta X_i^2}{2(\sigma_B\Delta t + \sigma_H)}\right).$$

Combining these the probability is obtained to observe X_{i+1} if there is a value of X_i a step earlier. This is the probability of observing no jump multiplied by the probability for a change in X in the case of no jump, plus the probability for a jump times the probability for a change of X in the case of a jump. That is

$$\begin{aligned} \mathbb{P}[x_{i+1}|x_i] = & \frac{\exp(\lambda\Delta t)\lambda\Delta t}{\sqrt{2\pi(\sigma_B\Delta t + \sigma_H)}} \exp\left(-\frac{\Delta x_i^2}{2(\sigma_B\Delta t + \sigma_H)}\right) \\ & + \frac{1 - (\exp(\lambda\Delta t)\lambda\Delta t)}{\sqrt{2\pi\sigma_B\Delta t}} \exp\left(-\frac{\Delta x_i^2}{2\sigma_B\Delta t}\right). \end{aligned} \quad (4.3)$$

Algorithm 3 presents our Matlab pseudo code for our Likelihood.

Optimization of the likelihood function The likelihood function is derived which is required to be optimised in the previous section. To perform the optimisation, multiple approaches seem feasible. The easiest way might be to simply lay a grid over the parameter space, which is three dimensional since there are three parameters σ_B , λ and σ_H . This is, however, quite a slow process and not precise.

An alternative is to use numerical routines for finding maxima. The Matlab `fmincon()` function was used to perform this task and to reduce the computation time compared to the grid approach. `fmincon` is a Matlab program that finds the minimum of a given function `f` under constraints. It finds the minimum in a region of the space that is defined beforehand (e.g. by declaring that certain variables should not exceed a value of `x`).

Algorithm 3 Logarithmic Likelihood of a Dataset – Pseudo Code (Matlab)

```

1: Input  $Data, \Delta t, \sigma_B, \sigma_H$  and  $\lambda$ : Data contains the values  $x_i$  for which the likelihood is required,  $\Delta t$  is the step size,  $\sigma_B$  the
   scale of the Brownian motion and  $\sigma_H$  the jump scaling.
2: Output DatasetProbSum: The output is the logarithm of the likelihood of the observed data given the model parameters
    $\Delta t, \sigma_B, \sigma_H$  and  $\lambda$ .
3:  $DataSetProb = \text{zeros}(1, \text{length}(Data))$  % In this variable the probabilities are stored to observe the different  $x_i$ .
4:  $PoissonProb = \exp(-\lambda * \Delta t) * \lambda * \Delta t$  % This is the probability that a jump occurs between two time steps.
5: for  $i = 1 : \text{length}(Data)$  do
6:    $DataSetProb(i) = \text{normpdf}(Data(i), 0, \sigma_B * \Delta t + \sigma_H) * PoissonProb + (1 - PoissonProb) * \text{normpdf}(Data(i), 0, \sigma_B * \Delta t)$ 
7:   % This is equation 4.3 in Matlab code. The Matlab function normpdf calculates the Density of the Normal distribution at a
   given point.
8: end for
9:  $DatasetProbSum = -\text{sum}(\log(DataSetProb))$  % The logarithmical likelihood is calculated to avoid too small num-
   bers. The Matlab sum command sums up the values in a vector and the Matlab log command takes the logarithm.

```

Link to the HMM Interpretation

The probabilities were not specified in the HMM to transit into the jump state or into the no jump state. Furthermore, the standard deviations of the two Gaussian distributions were not stated and these can now be done based on likelihoods.

Both Gaussian distributions have a zero mean, the left (no jump) Gaussian distribution has a standard deviation of $\sigma_B \Delta t$ and the right (jump) Gaussian distribution has a standard deviation of $\sigma_B \Delta t + \sigma_H$.

The probability for a jump, i.e. $P(\text{"Jump"})$ in Fig. 4.3, is $\exp(-\lambda \Delta t) \lambda \Delta t$ and the probability for no jump, i.e. $P(\text{"No Jump"})$, is $1 - \exp(-\lambda \Delta t) \lambda \Delta t$.

4.3.4 Dependent Model

In the real world it is usually the case that the different companies are not independent of each other. For example, a global event like an increase in oil price, will hit a whole sector and not only a single company. Therefore, to make the model more realistic it is important to introduce a way to make the Bond prices of the different companies dependent to a common factor.

A natural way to do so is to introduce an object with which to represent global factors that influence the companies. This is the market trend and it is introduced in this section a model, which will be called the *dependent model*, that introduces dependencies between companies with the help of the market trend.

For simplicity the market trend is modelled with a scaled Brownian motion with the scaling adjusted σ_M to fit the market trend data as described in Section 4.2.

Another important observation is that not all companies are equally dependent on the market trend: some companies follow the market trend very closely while others are nearly independent of it. The correlation coefficients in Table 4.2 show how strongly the different companies are related to the market trend and these correlation coefficients are used in the model to define how strong the companies are coupled to the market trend.

Overall, in the dependent model the Lévy process of a company is changed to a conditional Lévy

process, i.e. a Lévy process that depends on the market trend simulated through a Brownian motion. This allows the to generation of correlated data from companies.

The parameters of our Brownian motion (equation 4.4) part are defined as:

- X is the increment in the trend.
- Y is the increment of a company due to the Brownian motion part.
- x is the observed increment of the market trend. This increment is denoted with ΔM .
- ρ is the correlation between the trend and the company as in Table 4.2.
- The means m_X and m_Y are both 0 as a Brownian motion is a zero mean Gaussian process.
- σ_X is the scaling of the trend, which is $\sigma_M \Delta t$.
- σ_Y is the scaling of the Brownian motion part of the company model, that is $\sigma_B \Delta t$.

In next sections, first the details of the parameteric model are described and then the inference procedure.

Hidden-Markov Model Interpretation

Similarly to the independent model the dependent model can be interpreted as a HMM model. Figure 4.6 illustrates the HMM. In the top row multiple companies are shown with each company having a HMM model similar to the HMM from the independent model in Section 4.3.3. The only difference is that the Gaussian distributions are also influenced by the trend which is shown in the lower part of the Figure 4.6.

The trend itself has only a single state as there are no jumps in the trend process. Based on the increment in the trend the increments in the different company models are changed. The corresponding Gaussian distributions for the companies are the ones in equation 4.5 and equation 4.6.

Parametric model

The market trend ($X_t, t \geq 0$) is modelled as a scaled Brownian motion, that is, it is a Gaussian process with mean 0 and covariance $\sigma_M \min(t, s)$.

Like in Equation 4.2 the likelihood to observe a given sequence can be split into the product of the likelihoods to observe the different increments. The likelihood for observing an increment of ΔX_i at step i is

$$\frac{1}{\sqrt{2\pi\sigma_M\Delta t}} \exp\left(-\frac{\Delta X_i^2}{2\sigma_M\Delta t}\right).$$

As in Section 4.3.3, the companies are again modelled as a Lévy processes. The difference in this section is that the increments are now dependent on the market trend. Both the Brownian motion increment and the Poisson jumps are made dependent on the trend.

The likelihood is for incremental changes through this from the standard Gaussian distribution to a conditional Gaussian distribution, i.e. conditional on the increment in the trend.

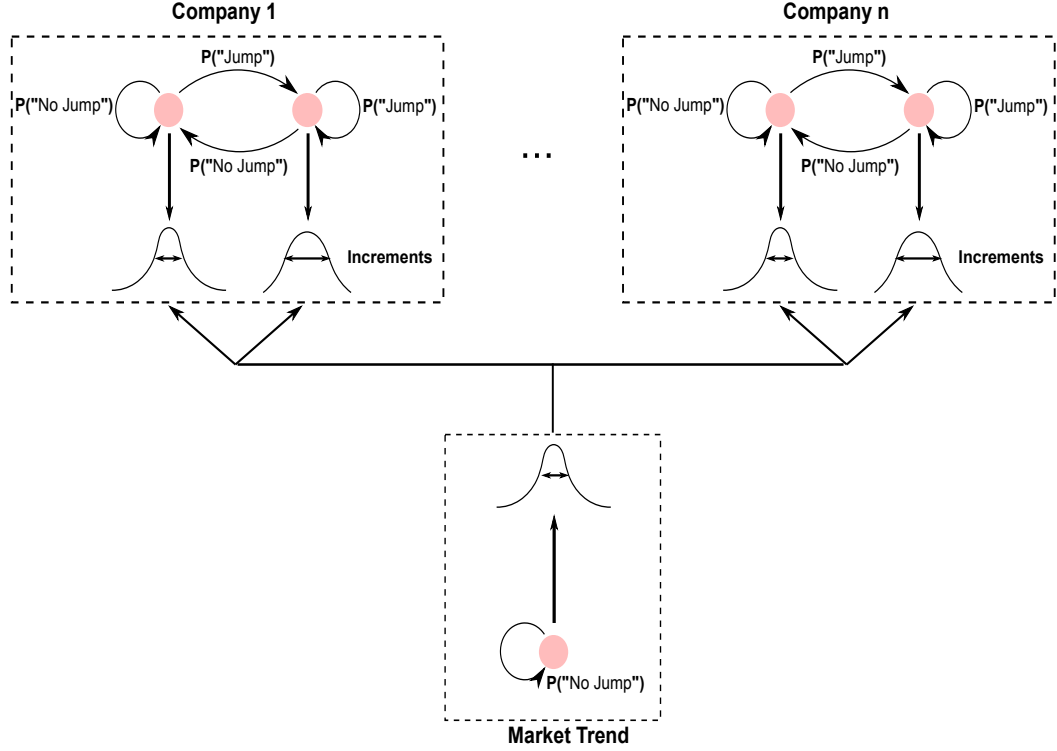


Figure 4.6: Conceptual Visualization of the Dependent Model

The conditioning of one Gaussian variable Y (here the increment of the bond price of a company) with another X (the trend) has the general form

$$\mathcal{N}\left(m_Y + \frac{\sigma_Y}{\sigma_X}\rho(x - m_X), (1 - \rho^2)\sigma_Y^2\right), \quad (4.4)$$

where \mathcal{N} is used to denote the Gauss distribution with the first entry being the mean and the second the variance of the conditioned Gaussian variable. m_X and m_Y are the means of the Gaussian, σ_Y and σ_X are the standard deviations and ρ is the correlation strength between the two variables. Finally, x denotes here the observed increment on which it is conditioned. In this setting this will be the increment in the trend and ρ the covariance from Table 4.2.

The equation is intuitive: For a high correlation, i.e. a high ρ the mean increment of the company is dominated by the increment in the trend (that is the $\rho(x - m_X)$ part). Furthermore, the higher the correlation ρ the lower is the variance $(1 - \rho^2)\sigma_Y^2$ of the conditional variable. This is also intuitive. If both variables are perfectly correlated, i.e. $\rho = 1$ then the increment in Y is simply the increment in X and there is no randomness, i.e. no variance, remaining.

This equation can now be applied to this setting. The two quantities that are changed compared to the independent model are the increment of the Brownian motion part and the increment due to the Poisson jump parts.

Using equation 4.4 together with these quantities the distribution for the increment based on the Brownian motion part is obtained:

$$\mathcal{N}\left(\frac{\sigma_B}{\sigma_M}\rho\Delta M, (1 - \rho^2)\sigma_B^2\Delta t^2\right). \quad (4.5)$$

In the same way the distribution for the increment based on the jump part of the Lévy process is obtained. The only thing that changes compared to the above quantities is the standard deviation σ_Y which is now simply σ_H , the parameter for the jump height distribution. Applying these definitions to equation 4.4 we get

$$\mathcal{N}\left(\frac{\sigma_H}{\sigma_M \Delta t} \rho \Delta M, (1 - \rho^2) \sigma_H^2\right). \quad (4.6)$$

Parameter inference

Like in the independent model maximum likelihood to infer suitable parameters is used. The Matlab routine *fmincon()* is applied to optimise the likelihood function. The difference from the independent model is that first the market trend parameter σ_M is optimised. This market trend parameter is then used with the trend data from Section 4.2 to perform maximum likelihood inference on the parameters of the 20 companies. By deriving first the parameters of the company the maximum likelihood inference for the 20 companies could be split as each company is independent in this model from the other companies, given the market trend.

4.3.5 Results

In Table 4.5 the inferred parameters for the our 20 companies are presented where the first row indicates the companies indexes (e.g. 1, 2, 3, ...) and the first column indicates the parameter's name (e.g. λ , σ_H , and σ_B). The dataset type is also mentioned in the table: **normal time** and **crisis time**. For example, the parameter λ of company number 1 has a value of 0.2 in both normal time and crisis time, however, the company 1, σ_H value increases from 0.499 in normal time to 1.283 in crisis time. This simply shows that company 1 has bigger spikes in the crisis period. These parameters were inferred with the help of the maximum likelihood procedure. The underlying model in this case was the independent model.

Table 4.5: Extracted Parameters for Independent Model

Company	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Normal Time																				
λ	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200
σ_H	0.499	0.470	0.490	0.436	0.447	0.557	0.494	0.507	0.454	0.497	0.449	0.480	0.466	0.447	0.461	0.460	0.498	0.483	0.490	0.451
σ_B	0.195	0.193	0.197	0.194	0.193	0.187	0.190	0.194	0.186	0.193	0.192	0.196	0.197	0.194	0.194	0.189	0.194	0.193	0.192	0.195
Crisis Time																				
λ	0.200	0.200	0.200	0.132	0.200	0.200	0.200	0.082	0.200	0.153	0.200	0.200	0.200	0.200	0.200	0.200	0.114	0.200	0.200	0.200
σ_H	1.283	1.303	1.345	1.830	1.563	2.217	1.408	6.028	1.842	3.961	1.463	1.414	1.230	1.807	1.071	1.603	2.990	2.115	1.373	1.490
σ_B	0.314	0.342	0.324	0.363	0.312	0.337	0.329	0.470	0.362	0.362	0.286	0.344	0.304	0.326	0.334	0.348	0.401	0.336	0.287	0.315

Figure 4.7 shows simulated bond prices for company number 1. Three paths are sampled from the independent model with the parameters from Table 4.5 for a period of 12.8 years (4680 data points). Each path includes four parts: two parts based on extracted parameters from the normal time period and two parts based on extracted features from the crisis period. The crisis periods are from data point number 1800 to data point number 2520 and from data point number 3600 to 4680, and are highlighted by purple arrows. As presented the simulated data (1,2 and 3) are similar to the real data in terms of jump height and rate of jumps. Since the increment of the company bond price is stochastic there are many potential realisations of how the bond price develops for the company and the chance that two curves

sampled from the process are identical is zero.

In general, the independent model is an efficient model to generate data for entities which are behaving completely independently of each other. That is, if it is known that company A has an increase (or decrease) in Bond price then this indicates nothing about how any other company B will perform.

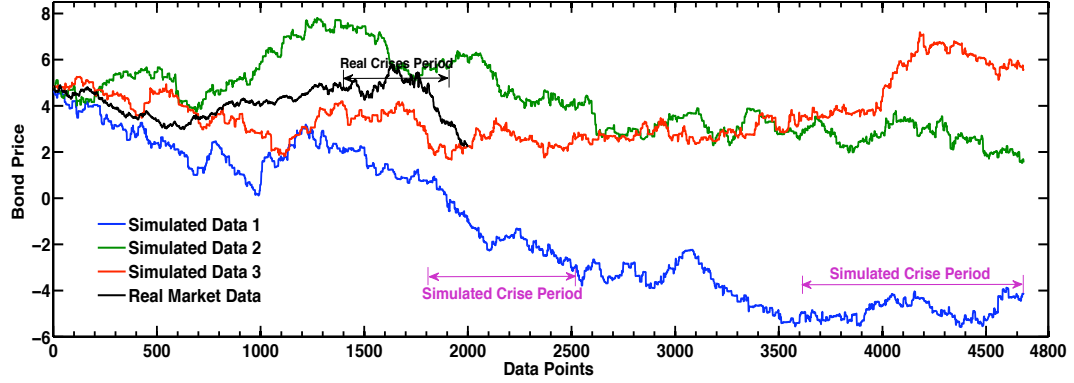


Figure 4.7: Simulated Data Vs. Company Data

Table 4.6 presents the parameters of the companies inferred with the maximum likelihood procedure under the dependent model. Similar to table 4.5, the first row of the table represents the 20 companies indexes (e.g. 1, 2, 3, ...) and the first column indicates the parameter's name (e.g. λ , σ_H , σ_B , and ρ). The dataset type is also mentioned in the table: **normal time** and **crisis time**. For example, the parameter λ of company number 1 has a value of 0.2 in both normal time and crisis time, however, the company 1, σ_H value increases from 0.363 (column 2, row 2) in normal time to 0.912 (column 2, row 6) in crisis time. Although there is still an increase in company 1 big jumps, but this value is decreased compared to the independent model. However, parameter λ shows that company 1 strongly (0.85) follows the market trend (column 2, row 4: 0.85) in normal time while its behaviour in crisis time shows little (column 2, row 8: -0.05) dependency on the market trend.

Comparing the entries with the corresponding entries from the independent model in Table 4.6 shows that the parameters differ between the two models. The reason for this difference is that the company is now behaving independently of the common market trend: if the market trend describes the company very well then there is little variation in the company behaviour left to describe and the variance parameters σ_H and σ_B become small. In the independent model all the variation must be described by these variance parameters and it is to be expected that the variance parameters in the independent model are higher than those in the dependent model for the majority of the companies. This intuition is consistent with the entries in the tables: the majority of σ_H and σ_B parameters is higher for the independent model.

Another important observation is that the correlation factor chosen by the maximum likelihood procedure differs from the correlation coefficients in Table 4.4 for the crisis period. This effect does not happen for the normal time period. The reason for this behaviour is that there are few observations (800) for the crisis time from which the parameters are inferred and the crisis period is a highly volatile period. It is not uncommon that inference faces problems in high variance, few data, settings and this effect is

expected to vanish if sufficient data is available.

Table 4.6: Extracted Parameters for Dependent Model

Company	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Normal Time																				
λ	0.200	0.001	0.092	0.200	0.200	0.200	0.001	0.200	0.200	0.155	0.200	0.164	0.200	0.200	0.103	0.200	0.005	0.200	0.200	0.200
σ_H	0.363	1.613	0.206	0.272	0.276	0.320	0.627	0.495	0.274	0.238	0.302	0.145	0.291	0.279	0.006	0.502	0.648	0.337	0.480	0.280
σ_B	0.242	0.271	0.273	0.000	0.000	0.000	0.278	0.198	0.000	0.265	0.000	0.269	0.000	0.000	0.270	0.000	0.282	0.232	0.194	0.000
ρ	0.850	0.950	0.950	0.950	0.950	0.900	0.950	0.150	0.950	0.950	0.200	0.950	0.950	0.950	0.950	-0.700	0.950	0.800	0.150	0.950
Crisis Time																				
λ	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200
σ_H	0.912	1.245	1.776	0.792	1.141	1.858	1.749	2.764	1.148	2.539	1.742	1.138	0.947	1.217	1.233	2.429	1.736	0.999	1.179	0.535
σ_B	0.300	0.000	0.000	0.303	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.299	0.000	0.000
ρ	-0.050	-0.900	-0.950	-0.100	0.850	0.800	-0.950	0.950	0.800	-0.950	-0.950	-0.850	-0.800	-0.900	-0.900	-0.900	-0.950	-0.050	-0.900	-0.150

Figure 4.8 illustrates the simulated bond prices for company number 1, correlated with the market trend. As shown, the company bond price mimics the trend behaviour for the normal period (first 10000 data points) while showing almost no similarity for the crisis period. This behaviour can be explained through the correlation values between the company and the trend which is $\rho = 0.85$ for the normal period and a very low correlation, $\rho = -0.05$, for the crisis period (see Table 4.6).

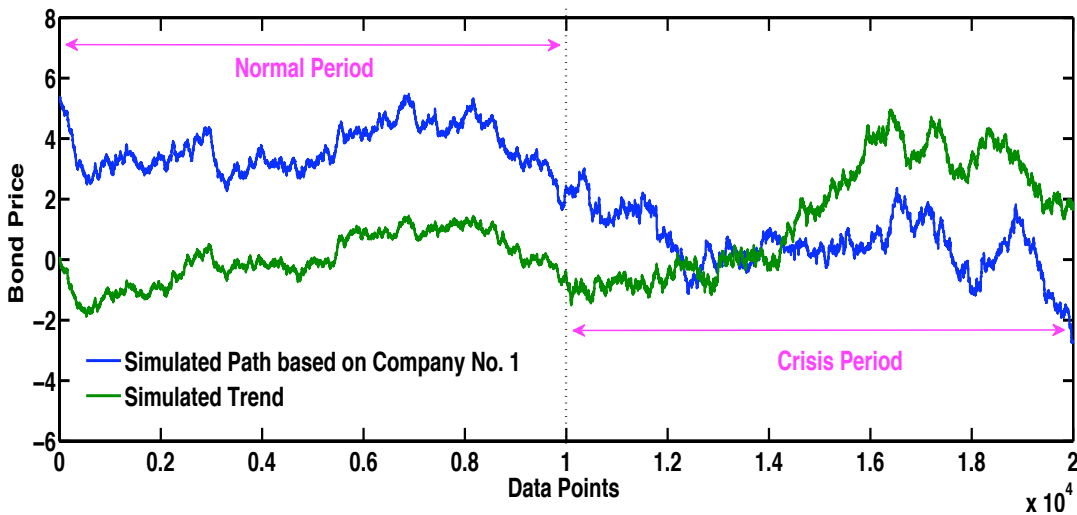


Figure 4.8: Simulated Data Based on Company Number 1 and Correlated with Market Trend

To better illustrate the advantage of incorporating the correlation factor in the data generative model, Figure 4.9 presents simulated data for company number 2, 3 and 19. These companies are intentionally selected since companies number 2 and 3 show a high correlation value ($\rho = 0.95$), while company number 19 shows a low correlation ($\rho = 0.15$) for the normal period. As pictured by Figure 4.9, by introducing the market trend into the generative model, the simulated datas are successfully enforced to share similarities in their behaviour. This is extremely useful when simulating entities which share similarities in their behaviour like, for example, companies from an industry sector that depends on oil price fluctuations, interest changes or other common factors.

The approach also allows us to study the company behaviour in different scenarios. Figure 4.8 and Figure 4.9 demonstrate this. The simulation underlying the bond prices shown in Figure 4.8 has two periods, a normal time period until datapoint 10000 followed by a long lasting crisis period until datapoint

20000 allowing the study of company behaviour in long lasting stock market crises. The simulation underlying Figure 4.9 has a different setup; here normal times are dominant and are interlaced with highly volatile but short crisis times. One can clearly see that these crisis times have a far stronger effect on the company bond prices than the normal times and company bond prices can rapidly increase or decrease in a short amount of time. Also companies that are highly correlated in normal times can show completely different behaviour in the crisis periods (as demonstrated by the red and blue curve).

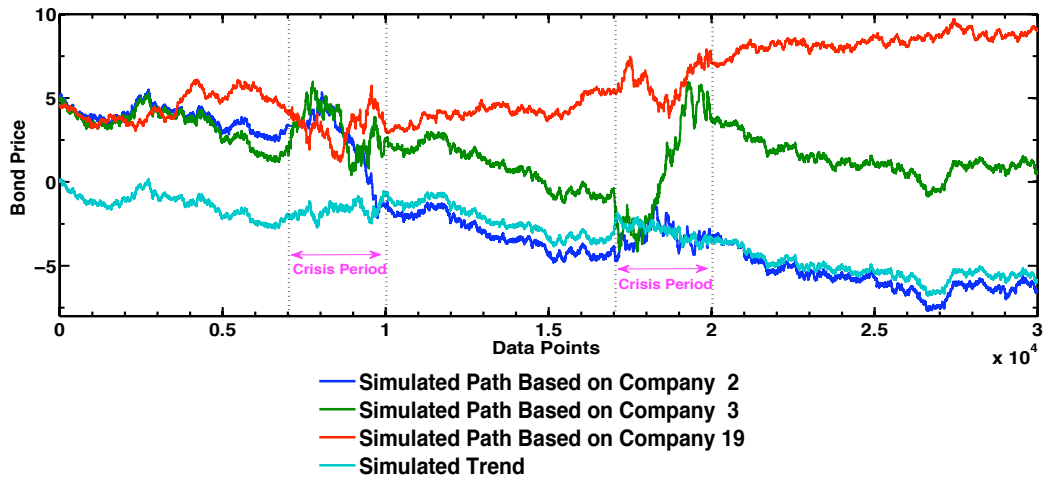


Figure 4.9: Three Simulated Data Based on 3 Companies and Correlated with Market Trend

4.4 Evaluation and discussion

The apparent question is now: is the effort to derive and implement the dependent model with Levy processes worth it? The question is best answered by comparing the approach with alternatives⁷. The possibly easiest alternative approach is to simply repeat the real data in a loop, ie, if the final time T is reached it restarts to generate the company data from time 1. This leads, obviously, to the most exact representation of the data, however, it does not capture and intrinsic properties of the process and the approach is questionable as a test bench. In particular, over-fitting will become an issue if this approach is used as a benchmark: A method to be tested could, for example, exploit this structure to get exact knowledge of the future and, hence, of the bond prices that will appear at any given future time. Now, if, for instance, the success of trading strategies is to be tested then a method that makes use of this strategy would beat any competitor, but would most likely fail in any other setting. The other disadvantage is that the approach does not allow any perturbation of the system. For example, the effect of reducing the dependency of a company to the market trend may be studied. With this approach there is no possibility to do so or to simulate any other perturbation of the system.

If a generative approach is used, that is, an approach that generates data based on some probabilistic law, then one approach is a time series that at each time step t changes its price according to a normal

⁷This model is not validated with reproducing the stylised facts (see Section 2.1.2 of [Martinez-Jaramillo, 2007] for list of stylised facts.), such as volatility clustering and heavy tails, since Lévy process is a well-established technique in computational finance. However, this could be further investigated in future studies.

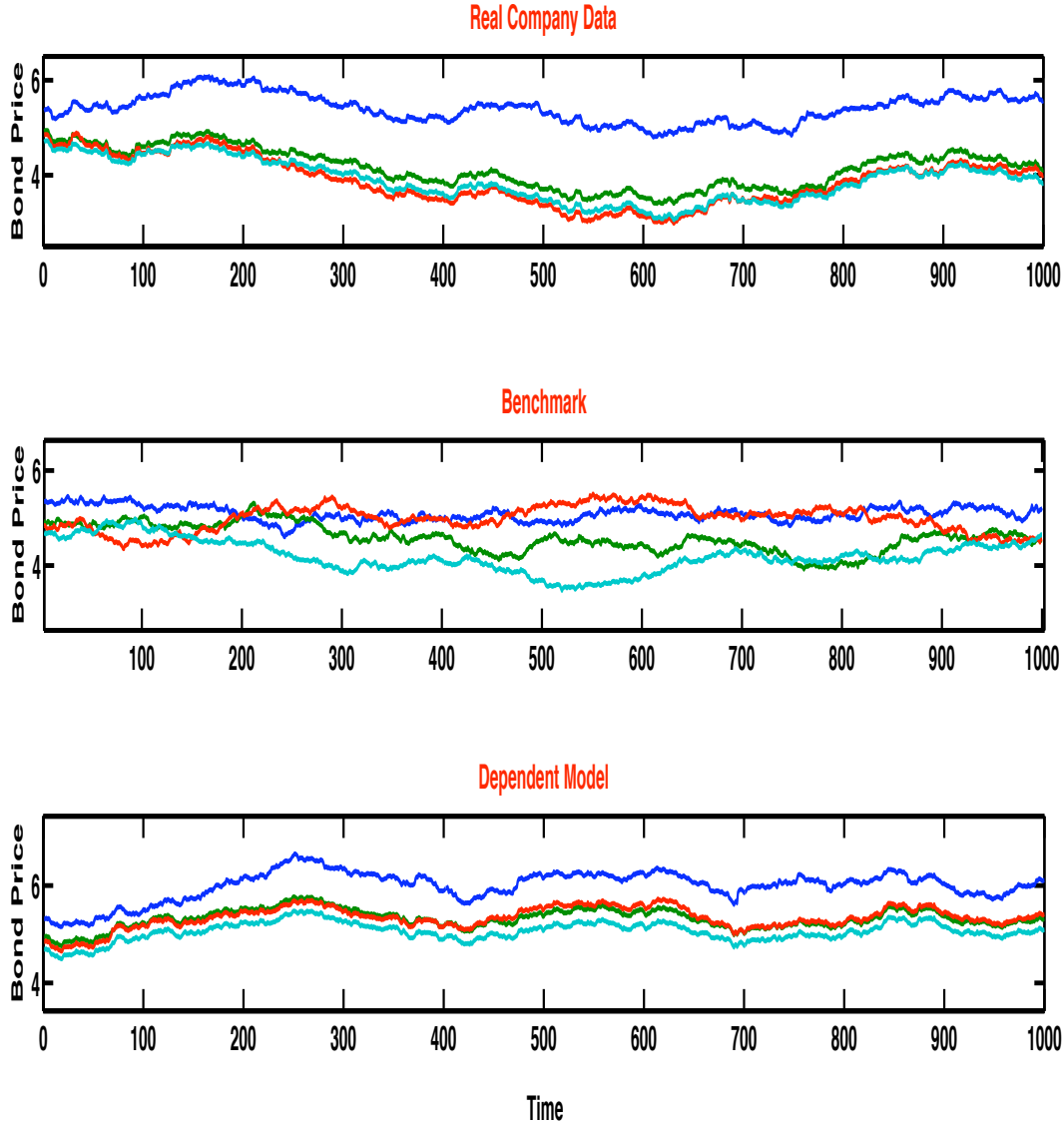


Figure 4.10: Validation

distribution. This model is called an Autoregressive (AR) model and it is widely used to simulate and study time series data. The model is also equivalent to a simple Brownian motion. This approach is, hence, closer to our approach. The two differences are that rare events may be simulated with the help of the Poisson process part and, more importantly, that with this approach the bond prices of companies can be coupled. This is crucial to study the interplay of different companies with the market. Figure 4.10, shows a comparison of this approach to the real data and the AR approach. The top plot illustrates the company bond price for the first 4 companies of the data set from time $t=1$ to $t=1000$. In the middle bond price simulations are plotted from the dependent model for these 4 companies, and, in the bottom, data the AR model fitted to the company data are plotted. It is apparent that this approach captures the interplay between the four different companies, while the alternative approach captures only the individual behaviour but not the interplay between the companies. This effect can also be verified in numbers. Table 4.7 shows the correlation between the four companies for the real data, the generative model, the

benchmark model and, in the last two columns, the difference between this approach and the real data and the difference between the benchmark approach and the real data. For instance, second row of the table presents the correlation coefficient between the company 1 and 2, using real data (0.85), simulated data by dependent model (0.87), and benchmark technique (0.19). Column 6 and 7 are presenting the difference between the real data and dependent model (0.02), as well as the difference between the real data and benchmark approach (0.66).

Table 4.7: Correlation between Companies

	REAL DATA	DEPENDENT MODEL	BENCHMARK	REAL – DEPENDENT	REAL – BENCHMARK
Companies 1 and 2	0.85	0.87	0.19	0.02	0.66
Companies 1 and 3	0.82	0.88	-0.51	0.06	1.33
Companies 1 and 4	0.84	0.88	0.54	0.04	0.3
Companies 2 and 3	0.98	0.94	-0.34	0.04	1.32
Companies 2 and 4	0.99	0.99	0.52	0	0.47
Companies 3 and 4	0.98	0.94	-0.78	0.04	1.76

Clearly, this approach is reproducing the correlation and dependence structure much better than the benchmark AR approach. Therefore as illustrated by this result, the data generative approach is a valid contribution to study the interplay between companies and the market, improving upon the benchmark and allowing for perturbations of the system.

4.5 Conclusion

Two models have been derived to describe company bond prices: the independent model which considers each entity independently from the other entities and the dependent model in which the entity behaviour is linked through a common factor to other entities.

Lévy process was developed for both models which were fitted with the help of Maximum likelihood inference to adjust the model parameters to the real world data. As presented in section 4.3.5 both models successfully extract the features of the real market data and simulate the artificial data based on these features thus enabling open ended simulation with realistic artificial data. This is an important contribution for ACE community by introducing the real data features to artificial data hence bringing the AB simulations one step closer to reality. Our generating model can simulate any type of data as long as a reasonable amount of real historical data is available. The small amount of real data is equivalent to low accuracy when extracting the data parameters.

The next chapter is focused on the second challenge: Credit Default Swap (CDS) Pricing Technique where a novel approach is introduced which provides results to illustrate the practicability and validity of this approach.

Chapter 5

CDS Pricing Techniques

The world economy has faced one of the biggest crises ever seen in 2007-2010, throwing most countries into recession (see Section 2.1). The causes of the financial meltdown are numerous, but it is widely accepted that one significant factor was the Credit Default Swap (CDS) market. Trading of this complex financial product was unpredictable, out of control, and badly priced, leading to fortunes being made and lost [Anon, 2008]. Poor pricing of the CDS contracts was one of the major reasons which contributed to the financial crisis. Even though the CDS contracts were supposed to represent a kind of insurance against a loan default, their prices often showed little relationship to the true ability of the companies to repay those loans. Thus when companies unexpectedly defaulted on their loans (or unexpectedly paid the loans back), a bank that bought a CDS contract at a very high price might suddenly find out that the contract is actually worth very little and the bank would lose a considerable amount of money. Thus the pricing of CDS contracts is of enormous concern and consequence.

As explained in Section 3.1.2, the CDS price fluctuations highlights the company's financial situation and consequently its default risk. Therefore, the true pricing of the CDS contracts helps traders to have a better understanding of companies financial health thus helping companies to make better decisions. However, the exact pricing of CDS contracts is a quantitative and qualitative challenge which includes an estimation of the probability default, the timing of a default and balance sheet value fluctuations (see [Hull and White, 2000, Joro et al., 2004, Anon, 2008]). In Section 2.4, it was explained how the CDS contract can be priced using two well established models: the probability model and the no-arbitrage model. However, it was argued that the probability model enforces a considerable level of complexity and the no-arbitrage model fails in practice due to its existing arbitrage channel. This chapter is focused on addressing this challenge by providing an accurate CDS pricing technique without enforcing the unnecessary complexity of the Agent Based (AB) model. In this thesis, the Duffie and Hull approach is used as a theoretical CDS pricing benchmark.

$$CDS_{spread} = BondYield - InterestRate \quad (5.1)$$

This work is the first ever study performed of CDS pricing by using Cartesian Genetic Programming (CGP) to analyse the relationship between price, debt and equity information. This price discovery tool will be used later in our agent-based model. The advantage of having a dynamic price discovery tool

compared to using a passive pricing function is that the traders (agents) will be able to use a customised pricing function for each company. This is far beyond the conventional economic theory where all companies are treated in the same way. In the real world, each market factor affects each company in different ways based on the nature of the company and also the increase or decrease of each market factor (e.g. the interest rate) might affect a company in a different way. This tool will help traders to produce a tailored function for each company and also for any specific time period if it is needed.

In recent years, the CGP became increasingly popular due to its efficiency and flexibility compared to other forms of Genetic Programming (GP). Although CGP has attracted much attention from the evolutionary community as a regression tool it is not yet being compared to competitors in other fields. Regression is the study of relationships among variables with a primary intention of prediction or estimation of the value of one variable from values of other known variables related to it. We accomplish our study by investigating the CGP performance, efficiency and utility in comparison to a well-known statistical regression technique, Gaussian Process Regression (GPR), for the purpose of financial price discovery.

This chapter consists of four sections. Section 5.1 focuses on our CGP approach and it provides our motivation and the technical details. The results are also discussed in this section alongside our experimental settings and objectives. Section 5.2 presents our motivation for using GPR and explains its technical details. The experimental setting and results are also demonstrated. Section 5.3 will discuss our approach validity by means of experiments and comparison. The conclusion is provided in Section 5.4.

5.1 Cartesian Genetic Programming: Motivation

In 1954, GP began when a Norwegian-Italian mathematician, Nils Aall Barricelli, first applied Evolutionary Algorithms (EA) to evolutionary simulations [Barricelli, 1954]. GP is an Evolutionary Computation (EC) technique that automatically solves problems without having to tell the computer explicitly how to do it [Poli et al., 2007]. CGP is a highly efficient and flexible form of GP that encodes a graph representation of a computer program. CGP was invented by Julian Miller in 1999 and was developed from methods developed for the automatic evolution of digital circuits [Miller and Tomson, 2000] for the purpose of evolving digital circuits.

CGP has been applied to a growing number of domains and problems such as digital circuit design, digital filter design, image processing, artificial life, bio-inspired developmental models, evolutionary art and has been adopted within new evolutionary techniques such as cell-based optimisation and social programming [Miller, 2011]. It has not been investigated in the financial field so far while GP is widely used such as for: stock markets, game theory, betting, foreign exchange, arbitrage and studying markets. See [Poli et al., 2007] for a comprehensive overview of GP and its applications. One of the common CGP usages involves extracting the mathematical relationship between inputs. The positive results of previous work in different fields and problems prove the ability of CGP in working on opaque and ill-defined applications. In addition, having fewer limitations in terms of the number of inputs and outputs it can produce and the complexity of solutions it can generate compared to other approaches,

make this technique a good choice for a potentially human-competitive analysis tool for finance.

5.1.1 CGP Technical Definition

In general, CGP represents computational structures such as mathematical equations or computer programs as a string of integers. These integers, also known as genes, determine the functions of nodes in the graph, the connections between nodes, the connections to inputs and the locations in the graph where outputs are taken from [Miller and Tomson, 2000]. Using a graph representation is very flexible as many computational structures such as Artificial Neural Network (ANN)s can be represented as a graph thus they can be encoded easily in CGP. Recent published results of Khan and Miller represent how using CGP to encode and evolve ANNs is highly efficient and competitive with other methods of evolving ANNs [Khan et al., 2011]. Unlike traditional GP, CGP represents a program as a directed graph that for feed-forward functions is acyclic. The significance of the difference between CGP and Linear GP has been established as the means of restricting interconnectivity of nodes [Wilson and Banzhaf, 2008]. In CGP, the genotype is a fixed length representation and consists of a list of integers, which encode the function and connections of each node in the directed graph [Walker et al., 2006].

The number of nodes in the graph is bounded but it can be varied, as CGP uses a genotype phenotype mapping that allows the existence of unconnected nodes in the genotype which produce inactive sub-genotypes that have no effect on the phenotype. This leads to an effect on search called neutrality, a CGP feature that has been found to be tremendously valuable to the evolutionary process on the problems studied [Walker and Miller, 2004]. Each of the nodes is encoded by a number of genes representing a particular function and the inputs that each node has. The nodes take their inputs in a feed forward manner from either the output of a previous node or from one of the initial program inputs (terminals). The model inputs are numbered from 0 to $n + m - 1$ where n is the number of model defined inputs and m is the number of user defined inputs. These numbers are used for referencing the outputs of the nodes and the initial inputs of the program. If the problem requires k outputs then these will be taken from the outputs of the last k nodes in the chain of nodes. Figure 5.1 shows a genotype sample which has been evolved by CGP for the one input symbolic regression problem called polynomial koza-quintic¹: $Y = X^5 - 2X^3 + X$.

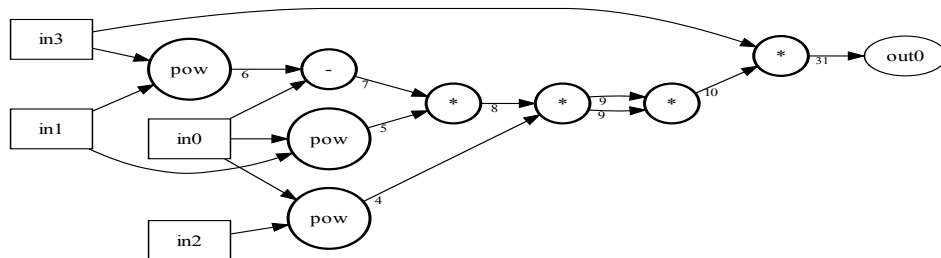


Figure 5.1: CGP Genotype Example Evolved by the Authors.

¹We use our chromosome translator which is designed in a recursive manner to transform a CGP solution to a mathematical notation.

Our CGP Model

We employ the CGP technique as a regression tool to learn and identify the best relationship between bond yield and interest rate in order to estimate the CDS price accurately. The classical CGP model only works with training set and it does not consider reporting on test set and the final solution is being saved in chromosome format. Moreover, it does not allow the actual outputs (e.g. CDS price) comparison as it only reports on fitness of the solution which represents the error rate of the whole data set.

For the purpose of fulfilling this research objectives, the basic CGP model² is modified in order to provide more information from the CGP result. This CGP model has the following features:

1. **Training/Test Dataset:** This version of CGP divides the data X_a into two datasets where X_i randomly chosen data points are considered as the test set and the X_{a-i} remaining data points are considered as the training set. The training set is used by CGP as input data, and the best result at the end of the evolutionary run is tested.
2. **CGP Output:** The original CGP model provide a fitness report as the CGP output. For the purpose of analysis this version of CGP reports on training and test sets results which make the data comparison possible.
3. **Chromosome Translator:** An important feature of this work is the evolved equation (not just the fitness values). Hence we have also created a solution parser that translates the chromosome into an understandable mathematical equation, which we can then study for insights into the solution.

5.1.2 Experimental Datasets, Settings and Objectives

The Centrica Plc company is chosen for our experiments. The main reason for choosing Centrica Plc over the full database is the availability of its equity information for the requested period at the time. The equity information is required for the purpose of studying one of the objectives.

Centrica plc is a large multinational utility company. It is listed on the London Stock Exchange and also listed on FTSE 100 Index³. Data is collected from 5th January 2004 till 25th June 2009, which includes the recent highly turbulent nature of the markets. Table 5.1 illustrates a sample of the dataset including the company CDS spread, debt and equity information. Two datasets are specified for the system. The first dataset contains three inputs: CDS spread (bp), bond yields (ask and bid price) and Bank of England base rate. The second dataset includes the all available information, eight inputs as shown in table 5.1. In the rest of this section we refer to the first dataset as the CDS-Debt dataset and the second dataset is called CDS-Debt-Equity dataset. Each dataset contains 1400 data points (X_a) where 400 randomly chosen data points (X_i) are considered as the test set and the 1000 remaining data points (X_{a-i}) are considered as the training set. In choosing the sample size, a general rule is followed which says the training set must be large enough so the accuracy will not drop significantly, and the test set must be large enough to silence random fluctuations. In our case, the 2:5 ratio provided a more accurate result in comparison to other popular ratios (e.g. 1:10 test:train ratio is popular because it looks round,

²Visit <http://www.cartesiangp.co.uk> for CGP related information and CGP model.

³FTSE 100 index is a share index of the 100 most highly capitalised UK companies listed on the London Stock Exchange.

1:9 is popular because of 10-fold cross validation, 1:2 is popular because it is also round and reassembles bootstrap). The test set is the same for all runs.

Table 5.1: Centrica plc Database (CDS, Debt and Equity Information)

Date	Spread(bp)	Bid yield	Ask yield	Base rate	Bid PX	Ask PX	E. Volatility	E. Weight	E. PX(High)
05/01/2004	0.2900	5.448	5.376	3.75	102.908	103.408	7.8840	187.7381	188.81
.....
24/06/2009	0.6667	4.172	3.990	0.50	105.188	105.768	28.996	226.5237	230.75
25/06/2009	0.6652	4.157	3.972	0.50	105.232	105.822	28.840	227.5276	231.00

Table 5.2 shows the experimental setup. All experiments were run with the same settings. The mutation rates were varied and the number of nodes (which in CGP affects the overall size of solutions and thus the complexity of equations that can be evolved) in order to monitor CGP behaviour. A simple function set is chosen, containing only fundamental operators as listed in table 5.2. In addition to the financial inputs, three constant integers (1, 2 and 3) have been given as constant inputs to the model as well.

Through some initial experiments, the generation number of 200000 was shown to be sufficient as the model fitness did not improve with a larger number. The CGP model documentation also recommended a population size of 5 and a row number of 1 as this has been recommended for this CGP model. However, for the number of columns, two different numbers of 250 and 500 were chosen to allow monitoring of the CGP solution characteristics. Three different mutation rate of 0.20, 0.50, and 0.70 were selected to cover the maximum, minimum and average mutation rate. Due to the long CGP simulation time, it was not feasible to test the model with a variety of different settings. Therefore, we chose those settings which proved to be sufficient for our chosen datasets in the preliminary experiments.

Table 5.2: CGP Settings

General Setting	Function	Symbol
Population size: 5	Add	+
Mutation rate: 0.20, 0.50, 0.70	Subtract	-
No. of generations: 200000	Multiplication	*
No. of runs: 20	Division	/
No. of rows: 1	Power	Pow
No. of cols: 250 or 500	Square root	Sqrt
Levels back: 250 or 500		

The fitness is calculated for each data point by defining the error rate, calculated as the absolute value of difference between the CGP-Output and the actual data: $Error = |CGP_{Output} - Data|$ and converting this result to a number between 0 and 1 where this number demonstrates the portion of the number of actual values that is predicted correctly by CGP: $DataPointFitness = \frac{1.0}{1.0 + Error}$. The fitness of the whole dataset is equal to the sum of all data points' fitnesses and the best dataset fitness is equal to the number of data points. Thus, a higher fitness means a better result as it shows a smaller error rate. there are two main objectives in these experiments:

1. **Monitoring CGP behavior under different settings.** Therefore, the first experiments are run on the CDS-Debt dataset with different combinations of nodes (500,250) and mutation rates (0.20, 0.50, 0.70) to see how these two factors will affect the results. Following these experiments, the ability of CGP to deal with and distinguish between relevant and irrelevant inputs is examined by using the second dataset (CDS-Debt-Equity dataset) containing more data attributes.
2. **Assessing the CGP reliability as regression discovery tool.** Of interest is to observe whether CGP can come up with a regression model that can price CDS better than the regression benchmark model (Duffie Theory) and to help understand something of how that model works.

Each experiment is run 20 times to present the stability of the experimental results while ensuring that the required results are generated within the required time frame.

5.1.3 CGP Results

Figure 5.2 shows the CGP fitness report on CDS-Debt dataset. According to the result, although the number of nodes (graph C and D) and mutation rate (graph A and B) affect the CGP performance in terms of reaching a better fitness in early generations, it does not have a big impact on the average fitness. This means that the better fitness does not always rely on a larger number of nodes and higher mutation rate (graph B and C). For CDS-Debt dataset, the best fitness was achieved by 500 nodes and mutation rate of 0.50.

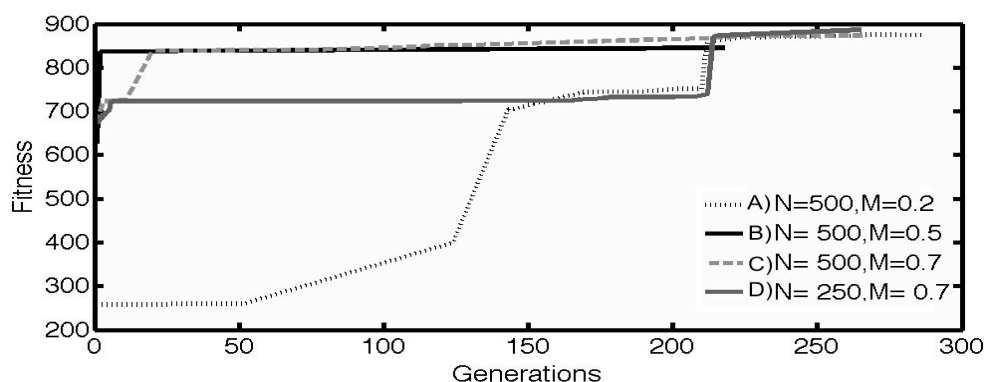


Figure 5.2: CGP Behavior Under Different Settings: N refers to number of columns and M stands for mutation rate in CGP settings.

As was discussed in Section 2.4, one of the important issues of CDS pricing is to reduce the arbitrage channel. Figure 5.3 and Table 5.3 show the results in terms of accuracy of CDS pricing. In these experiments, CGP discovered a new relationship between bond yield and risk free rate which creates a very accurate prediction of the real CDS price in the market. It also demonstrates that the arbitrage gap which exists in the Duffie theory is significantly reduced (see Figures 5.3(a) and 5.3(b)). The result shows that the trend of the CDS price has been predicted correctly.

In the experiment using the larger dataset (CDS-Bond-Equity dataset), the results show that the extra inputs helped CGP to reduce the arbitrage channel in some parts but it had a negative effect on other parts (see Figures 5.3(c) and 5.3(d)), so the overall error increased, see Table 5.3.

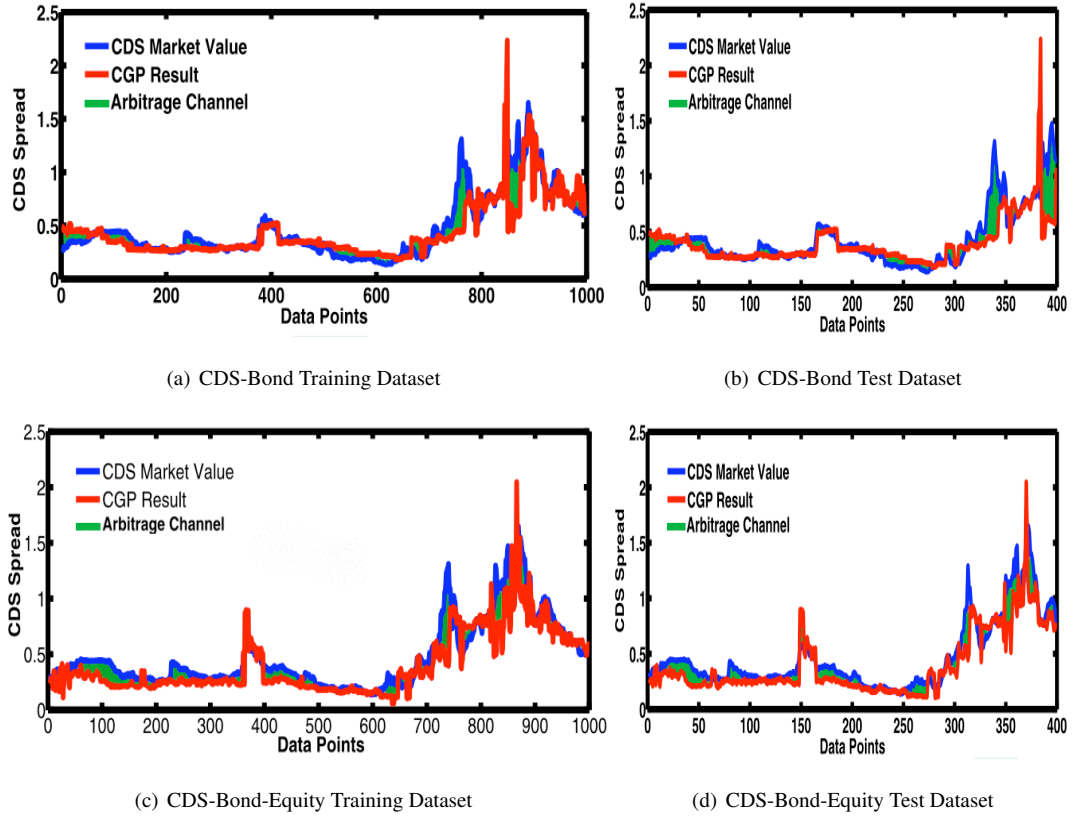


Figure 5.3: CGP Experiments Result

Table 5.3: Error Report

Model	Input	Training Set Error	Test Set Error
Duffie	CDS-Bond	167.84928%	134.6789%
CGP	CDS-Bond	8.9769295%	9.7533925%
CGP	CDS-Bond-Equity	10.1807889%	10.0130045%

The inability of CGP to perform effective feature selection using this larger number of attributes may be partly because of the complexity of this problem. Some of the additional variables may be useful some of the time and detrimental at other times, meaning that by using CGP (and indeed any evolutionary algorithm) it would be hard to eliminate them. The complexity of the relationships means that an incremental change from a complex solution using more variables into a simpler solution using fewer variables may be impossible without encountering extremely unfit variants, thus making the search unlikely to be successful. Nevertheless, the results are fascinating for they indicate that good accuracy for this problem can be obtained with fewer variables and simpler corresponding equations.

$$\text{CDSspread} = \frac{\left(\frac{-X_1^2 X_3}{3X_3} - 1\right) X_2 * (X_2 - X_3)}{\sqrt{\frac{X_2}{Z_1^3 X_1^2 * (-X_3)}} + (2 * D_5 \sqrt{\left(\frac{S_1}{Z_1}\right)^{D_6} * (Z_1 - M_1)})} \quad (5.2)$$

Where

$$\begin{aligned}
D_2 &= (4 - X_1) * (-X_3^{X_2-1} - M_1), \quad D_3 = X_3 - D_2 - 3^{X_3}, \quad M_1 = X_2^3 + \frac{X_2^3}{X_3} \\
D_5 &= \frac{D_3}{\frac{X_1^2}{X_1 - X_2 - 1}} * D_4, \quad D_6 = X_1 - \frac{3}{-X_1^2 X_3}, \quad D_4 = \frac{D_3}{Z_1 * 3^{\frac{3}{-X_1^2 X_3}}} * s_1 \\
S_1 &= X_2 * (X_2 - X_3)^2, \quad Z_1 = \frac{\frac{X_2^3}{X_3}}{X_1^2 - X_3}
\end{aligned}$$

Not surprisingly, CGP has evolved a completely different equation in each run. Equation (5.2) shows one of the best evolved solutions. Analysis of all evolved equations reveals that some components are repeated in all solutions. For instance the component $(\frac{X_2}{X_3})$ has been found in the 12 best solutions. X_2 is the buying price and X_3 is the selling price of a bond yield. Moreover, the component $(X_2 - X_3)$ which shows the difference between the sell price and the buy price of a bond yield is several regions of the equations of 7 best solutions. CGP has discovered these possible relationships between X_2 and X_3 (buy and sell prices). To understand the significance of these relationships, the effect of these two components are tested by reducing the difference between the sell price and the buy price.

$$\lim_{(X_2 - X_3) \rightarrow 0} \text{CDSspread} \quad \text{and} \quad \lim_{(\frac{X_2}{X_3}) \rightarrow 1} \text{CDSspread} \quad (5.3)$$

The computational results show that the error rate significantly increases when the difference amount between X_2 and X_3 is ignored but the theoretical regression benchmark ignored these relationships completely by using the average value of the buy and the sell price or just one of them.

5.2 Gaussian Process Regression: Motivation

Supervised learning can be divided into regression and classification problems. The outputs for classification are discrete class labels, while regression is concerned with the prediction of continuous quantities [Rasmussen and Williams, 2006]. For example, in a financial application, one may attempt to predict the price of a derivative such as a CDS as a function of interest rates, and bond price. In this section the Gaussian process methods for regression problems are described. The Gaussian process is a simple and general class of probability distributions on functions [Snelson, 2007]. Gaussian processes were first used for time series prediction in the 1940's [Wiener, 1949, Kolmogorov, 1941] and thereafter they have been widely used in the fields of geostatistics and meteorology (since the 1970's) [Snelson, 2007]. In geostatistics, prediction with Gaussian processes is termed kriging, named after the South African mining engineer D. G. Krige by Matheron [Matheron, 1973]. The Gaussian processes were then applied by statisticians to the slightly more general multivariate input regression problem (e.g. [O'Hagan and Kingman, 1978]).

GPR is one of the most widely used regression techniques and it achieved the top rank in an investigation by Robert R. Andrawis of stock returns prediction [Andrawis et al., 2011]. GPR is selected as the best model by the authors followed by neural networks, the multiple regression model and a simple moving average among the nine categories of models including: standard multilayer neural network,

Gaussian process regression, echo state network, echo state network ensemble, ARMA, AR, multiple regression, a version of Holt's exponential smoothing and simple moving average. GPR has numerous successful applications in different fields such as cellular networks [Schwaighofer et al., 2004], communication systems [Murillo-fuentes and Pérez-cruz, 2005], financial prediction [Chapados and Dorion, 2013], and robotics [Wu and Movellan, 2012]. This thesis is concerned with the use of Gaussian processes for prediction in a machine learning context. Williams and Rasmussen first described GPR in the machine learning context in 1996. The definitive book on GPR is a recent one by Rasmussen and Williams [Rasmussen and Williams, 2006]. The material covered in this section can be found there with more details.

5.2.1 GPR Technical Definition

In probability theory and statistics, a Gaussian process is a stochastic process whose realisations consist of random values associated with every point in a range of times (or of space) such that each such random variable has a normal distribution. Moreover, every finite collection of those random variables has a multivariate normal distribution [Simon, 1979, Rasmussen and Williams, 2006]. There are several ways to interpret GPR models. The simple linear regression model where the output is a linear combination of the inputs has been studied and used extensively. Its main virtues are simplicity of implementation and interpretability. Its main drawback is that it only allows a limited degree of flexibility; if the relationship between input and output cannot reasonably be approximated by a linear function, the model will give poor predictions [Simon, 1979].

GPR assumes that $f(x)$ relates to some specific models (*e.g.* $f(x) = mx + c$). A Gaussian process can represent $f(x)$ obliquely, but rigorously, by letting the data speak more clearly for themselves. Imagine there is a training set \mathcal{D} of n observations, $\mathcal{D} = \{(x_i, y_i) \mid i = 1, \dots, n\}$, where x denotes an input vector (covariates) of dimension D and y denotes a scalar output or target (dependent variable). The column vector inputs for all n cases are aggregated in the $D \times n$ design matrix ⁴ X , and the targets are collected in the vector y . Therefore, it can be written as $\mathcal{D} = (X, y)$ [Ebden, 2008]. In the regression setting the targets are real values. The point is to make an inference about the relationship between inputs and targets, i.e. the conditional distribution of the targets given the inputs. One way to couple inputs with targets is to assume that the targets are based on a functional relation $f(x)$ which is disturbed by some measurement noise ϵ . In the Bayesian framework one assumes that the function f is itself a random object that is drawn from a probability distribution of functions. For the GPR case one assumes that this probability distribution is induced by a Gaussian process such as:

$$f \sim \mathcal{N}(\mu(x), k(x, z)), \quad (5.4)$$

where $\mu(x)$ is the mean of the Gaussian process, $k(x, z)$ its covariance function and x, z are input vectors. For the simulation setting it is assumed that the Gaussian process has a zero mean, variance σ_n^2 and an additive measurement noise. Furthermore, it is assumed that the noise is independent and identically

⁴In statistics texts the design matrix is usually taken to be the transposition of the definition, but the choice is deliberate and has the advantage that a data point is a standard (column) vector.

distributed. Hence, the observation model is

$$y = f(x) + \epsilon \text{ where } \epsilon \sim \mathcal{N}(0, \sigma_n^2). \quad (5.5)$$

A common choice for the mean function $\mu(x)$ of the GPR is the constant zero (zero-mean) and an often used Gaussian covariance function is chosen which is defined as:

$$k(x, z) = \exp \left[\frac{-(x - z)^2}{2l^2} \right] \quad (5.6)$$

where l is a parameter which roughly describes how close it is expected that different data points will be. The Bayesian assumption can be used together with the Bayes rule to compute the posterior distribution over the function f after having observed training data. The Bayes-rule is:

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{marginal likelihood}}. \quad (5.7)$$

The posterior distribution is again a Gaussian with a posterior mean $\mu_{\text{post}}(x)$ and a posterior covariance $\text{cov}_{\text{post}}(x, y)$. The posterior equations can be found in [Rasmussen and Williams, 2006][p. 16]. They are for the posterior mean:

$$\mu_{\text{post}}(x) = \mathbf{k}(x)'(K + \sigma_n^2 I)^{-1} y, \quad (5.8)$$

where x is the input point at which an evaluate of the posterior mean is required, I is the identity matrix, y is a vector with the observations for the training points, $\mathbf{k}(x) := (k(x, x_1), \dots, k(x, x_n))$ is a vector where the covariance between the new input x and all training inputs x_i is calculated, K is the kernel matrix where $K_{ij} = k(x_i, x_j)$. The posterior covariance between points x and z is given by

$$\text{cov}_{\text{post}}(x, z) = k(x, z) - \mathbf{k}(x)'(K + \sigma_n^2 I)^{-1} \mathbf{k}(z). \quad (5.9)$$

In case the number of training inputs exceeds a few thousand, an exact inference would take too long. Therefore an approximation is used based on a low-rank plus diagonal approximation to the exact covariance to deal with these cases. The general idea is to use inducing points and to base the computations on cross-covariances between training, test and inducing points only. The Gaussian Processes is used for the Machine Learning toolbox⁵ [Rasmussen and Nickisch, 2011] for these simulations. This toolbox is based on the main algorithms from Rasmussen and Williams [Rasmussen and Williams, 2006]. The basic GPR algorithm as in Rasmussen and Williams [Rasmussen and Williams, 2006] is also presented in Algorithm 4 for completeness.

5.2.2 Experimental Datasets, Settings and Objectives

The data is presented in the same way as it was for the CGP experiments (see Section 5.1.2). A short overview of the dataset design is provided here. Table 5.1 demonstrates a sample of the database including the company CDS spread, debt and equity information. The original dataset is divided into two datasets called CDS-Debt dataset and CDS-Debt-Equity dataset for the purpose of the experiments as explained in Section 5.1.2.

⁵Documentation for GPML Matlab Code can be found at <http://www.gaussianprocess.org/gpml/code/matlab/doc/>

Algorithm 4 Gaussian Process Regression - Pseudo Code (Matlab)

```

1: INPUT: Data  $\mathcal{D} = \{(x_1, y_1), \dots, (x_n, y_n)\}$ , Parameter  $\sigma^2$ 
2: OUTPUT: Regression function  $f(x)$ 
3: Let  $k(x, y) := \exp\left(-\frac{(x-y)^2}{2l^2}\right)$  % We define  $k(x, y)$  to be the Gaussian covariance function.
4: for all  $i, j \leq n$  do
5:   Calculate  $K(i, j) = k(x_i, x_j)$  %  $K$  is called the kernel matrix and the entry  $K(i, j)$  is the distance between input  $x_i$  and  $x_j$  measure with Gaussian covariance function.
6: end for
7: Set  $y := (y_1, \dots, y_n)^\top$ ,  $k(x) := (k(x, x_1), \dots, k(x, x_n))^\top$  % All the observed values  $y_1$  to  $y_n$  are collected in one vector  $y$ . The function  $k(x)$  maps  $x$  to a vector for which entry  $i$  is the covariance between a possible input value  $x$  and the observed value  $x_i$ 
8: Calculate  $w := (K + \sigma^2 I)^{-1} y$  % A weight vector  $w$  is calculated for the regression function with the help of the Gaussian process regression equation.
9: Set  $f(x) := k(x)^\top w$  % the regression function is a vector product between the weight vector and the  $k(x)$  function.

```

The GPR model has two parameters that need to be set. The main “parameter” is the covariance function. Here the Gaussian covariance function was used as defined in Equation 5.6 and a factor $l = 1/2$ was used. The other parameter is σ_n for which 0.1 was used. These two values proved to be the most accurate for this dataset choice among the values of 0, 0.1, ..., 0.9, and 1. The two main objectives for these experiments are:

1. **Monitoring GPR behaviour under different settings.** Therefore, two datasets are presented to the model. The first dataset contains the inputs which are defined as the main market factors for CDS pricing. In the second dataset equity information is added. This extra data can be considered as noise in the system as it includes some unrelated data in regards to CDS pricing. It is significant to see how GPR deals with this extra information.
2. **Comparing GPR and CGP.** Of interest is here to observe their efficiency under different conditions and see whether it is reasonable to present both models to future traders as price discovery tools.

5.2.3 GPR Results

The result graphs include three sets of data: CDS market value, the CGP approach and the GPR approach results (see Figure 5.4). Regarding the first dataset (CDS-Bond dataset), the results show that the CGP technique has a lower error for both the training and test set. However, the average error for each data point is around 0.10 as can be calculated from Table 5.4. In terms of comparison, the average difference per data point between CGP and GPR results is 0.0088 for the training set and 0.0095 for the test set. According to these results, see Figure 5.5(a) and 5.5(b), both GPR and CGP techniques behave similarly and the error is related to the part of data which for which neither were capable of predicting a precise value. Therefore, the average difference per data point is negligible and both GPR and CGP techniques have behaved in the same way.

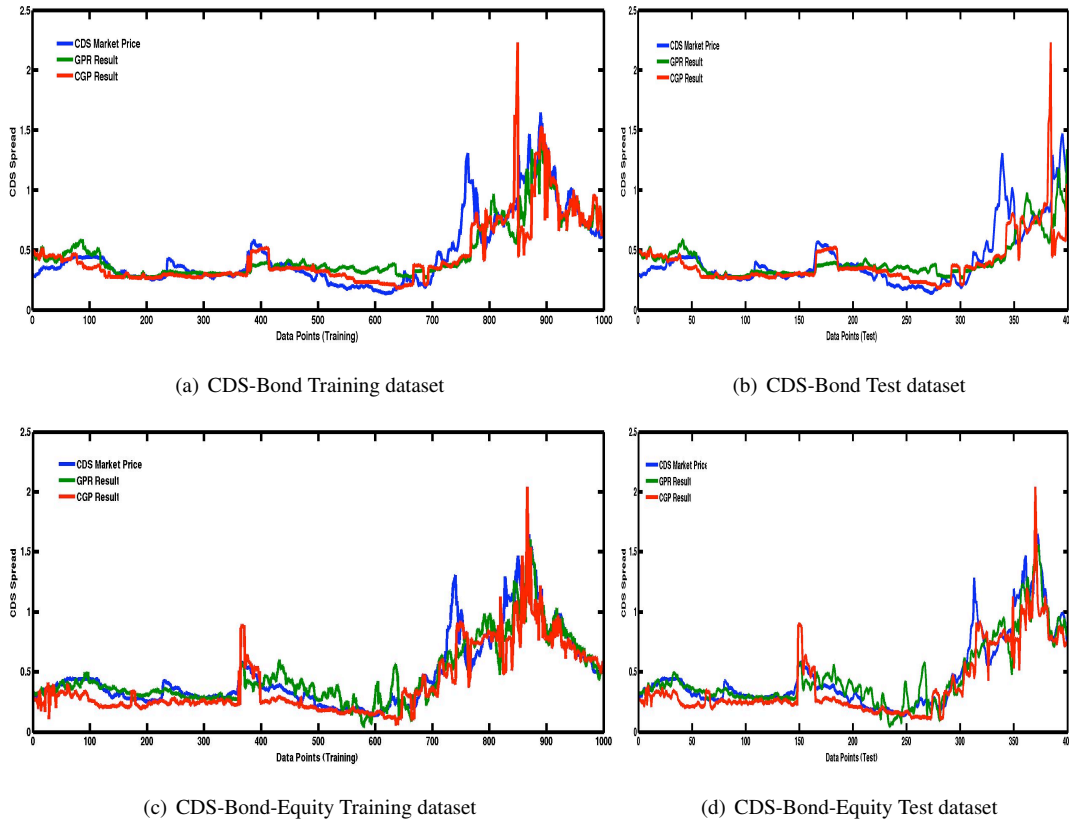
The results from the second dataset (CDS-Bond-Equity dataset) show that the GPR approach produced a better result in the noisy dataset. In terms of comparison, 5.4, the average difference per data

Table 5.4: GPR Vs. CGP (Error Report) for Centrica Company

Model	Input	Training Set Error (out of 1000)	Test Set Error (out of 400)
GPR	CDS-Bond	98.5323795	42.88902649
CGP	CDS-Bond	89.769295	39.01357
GPR	CDS-Bond-Equity	80.72867736	30.98058255
CGP	CDS-Bond-Equity	101.807889	40.052018

point between the CGP and GPR result is 0.0211 for the training set and 0.0226 for the test set. However, as shown in Figure 5.5(c) and 5.5(d), both techniques successfully predicted the trend (the same success as in the first dataset) and the major difference in their prediction result is in the first half of the training and test set. The figures show that in the first half of the dataset the GPR predicted a value which matches the CDS market value in most of the data points, while the CGP predicted a far lower price. Moreover, these results suggest that there is extra information in the CDS-Bond-Equity data set which the GPR has captured better than CGP.

Figure 5.4: GPR Experiments Result



5.3 Evaluation and Discussion

In Section 5.1 CGP was introduced as a regression technique to calculate the CDS price. The result demonstrated the capability of CGP as a financial price discovery tool. The CGP performance and accuracy were then investigated by applying the GPR approach (see Section 5.2). The GPR approach was suggested as an alternative to the CGP approach because the GPR technique is a well established

statistical regression tool which has been widely used in different fields but not in the field of Agent-Based Computational Economics (ACE). It was interesting to observe that the error rates are essentially the same for both techniques. However, these investigations and results have been based on one dataset including 1400 data points from Centrica thus it is not possible to generalise the result. The unavailability of requested data by the time of the investigation limited the study to Centrica.

In this Section, CGP and GPR techniques are used to find a best regression model between the CDS spread, bond yields and interest rates while a real-world comprehensive financial database is used including 40,000 samples from twenty iTraxx energy companies for a period of five years. The following section briefly describes the real-world dataset, experimental settings and objectives. This is followed by illustration of the results and implications.

5.3.1 Experimental Datasets, Settings and Objectives

Twenty companies from iTraxx⁶ energy have been chosen for these experiments. Data was collected from 1st January 2004 till 25th June 2009 (which includes the recent highly turbulent period of the markets). Figure 5.5 illustrates the bond prices of these twenty companies as well as interest rates for the mentioned period. The bond is the debt indicator of the company and interest rate will be used to assess the riskiness of the company situation based on its bond price.

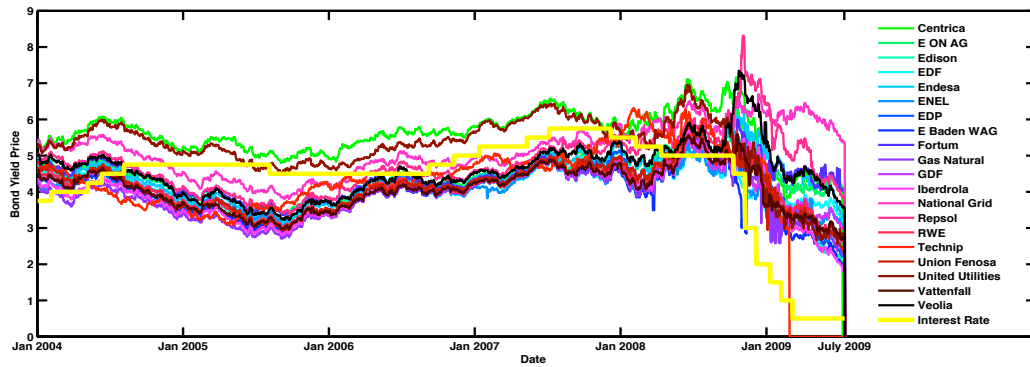


Figure 5.5: Real-world iTraxx Energy Data

Table 5.5: Original Datasetd

Date	Ir	BP_{ask}	BP_{bid}	CDS
01/11/08	3.75	5.38	5.30	0.2729
...
01/16/08	3.75	5.23	5.16	0.2850

Table 5.5 presents a snapshot of the main dataset including: date, interest rate (Ir), ask price of bond (BP_{ask}), bid price of bond (BP_{bid}) and CDS spread (CDS).

The cross-validation technique is used for defining both training and test datasets. Cross-validation is a technique for assessing how the results of a statistical analysis will generalise to an independent data

⁶iTraxx is the brand name for the family of credit default swap index products covering regions of Europe, Australia, Japan and non-Japanese Asia. They form a large sector of the overall credit derivative market.

set. It is mainly used in settings where the goal is prediction, and one wants to estimate how accurately a predictive model will perform in practice. One round of cross-validation involves partitioning a sample of data into complementary subsets, performing the analysis on one subset (called the training set), and validating the analysis on the other subset (called the validation set or test set). To reduce variability, multiple rounds of cross-validation are performed using different partitions, and the validation results are averaged over the rounds.

We use 5-fold cross-validation for selecting the training and test dataset among the 40,000 original samples. Five fold cross validation was used because it is a well-established method for verifying results [Refaeilzadeh et al., 2009]. The execution time of the system prohibited the use of more than five folds in this case. The original data sample is randomly partitioned into 5 subsamples. Of the 5 subsamples, a single subsample is retained as the validation data for testing purpose, and the remaining 5 – 1 subsamples are used as training data. The cross-validation process is then repeated 5 times so ensure that all data points have been introduced to the model for both training and test purposes, with each of the 5 subsamples used exactly once as the validation data. The advantage of the cross-validation technique over repeated random sub-sampling is that all observations are used for both training and validation, and each observation is used only once for validation.

The CGP model was used that was also used in Section 5.1. Table 5.6 demonstrates the experimental setup. All experiments were run with the same settings. The earlier investigation result was used as suggested to set the variables for this experiment. As presented in Table 5.6, the number of generations and the number of nodes vary in different experiments. A simple function set is chosen, containing only fundamental operators as listed in Table 5.6. In addition to the financial inputs, three constant integers (1, 2 and 3) were given as inputs to the model as well.

In this experiment, the mutation rate of 0.7 was chosen as it showed the same accuracy compared to a mutation rate of 0.5 and 0.2 while it improved the simulation time. The CGP model documentation also recommended a population size of 5 and a row number of 1 as this has been recommended for this CGP model. Two different numbers of 40 and 50 were chosen for the column sizes to reduce to the CGP solution size. However, the number of generations was increased to 150000 and 400000 to sustain the model accuracy. Due to the long CGP simulation time, it was not feasible to test the model with a variety of different settings. Therefore, those settings which proved to be sufficient for our datasets were chosen in the preliminarily experiments.

Table 5.6: CGP Settings

General Setting		Function	Symbol
Population size:	5	Add	+
Mutation rate:	0.70	Subtract	-
No. of generations:	150000, 400000	Multiplication	*
No. of runs:	15	Division	/
No. of rows:	1	Power	Pow
No. of cols:	40 or 50	Square root	Sqrt
Levels back:	40 or 50		

The same GPR model was used that was used in the earlier investigation and the model parameters were set as before (see Section 5.2 for full details of GPR approach). Here, the key parameters are briefly explained. The main “parameter” is the covariance function for which the Gaussian covariance function defined in Equation 5.6 is used and a factor $l = 1$ is used. The other parameter is σ_n for which 0.1 is used. These two values proved to be the most accurate ones for this dataset choice among the values of 0, 0.1, ..., 0.9, and 1. The overall objective of this investigation is to assess CGP as an efficient regression tool in comparison to GPR for the purpose of CDS pricing. The models were evaluated by focusing on two main objectives in these experiments.

1. **Comparing GPR and CGP.** Of interest, is to observe, their efficiency and accuracy as a price discovery tool as well as identifying the better approach.
2. **Monitoring CGP behaviour under different settings.** Therefore, the experiments were run with different combinations of nodes (40, 50) and different numbers of generations (150000, 400000) to see how these two factors would affect the results. Of interest is to observe whether it is possible achieve a better (in terms of the size of solution) solution by reducing the number of nodes and increasing the number of generations.

5.3.2 CGP Vs. GPR Results

Table 5.7 presents our experiment fitness result and error rate per data points for Duffie, CGP and GPR approaches. Note that in this experiment the number of nodes is set to 50 and number of generations is set to 400000 in CGP model. The winner is the approach which achieves a better fitness (less error rate per datapoint). The best fit for all dataset folds is 8000 (number of datapoints in each dataset). The results demonstrate the low accuracy of the Duffie approach while the CGP and GPR approach exhibited similar accuracy (see Datapoint Err in Table 5.7). The CGP model achieved a better fitness in three datasets (Fold no. 2, 3, and 5). However, the overall result of both approaches is very close and suggests the difference of 0.00094 unit per datapoint which is not considerable.

The estimated CDS price by CGP and GPR models alongside market CDS prices are illustrated in Figure 5.6 for each dataset fold. As suggested by these results, both the CGP and the GPR approaches successfully predicted the CDS price trend direction and also the CDS price most of the time. Although the 0.13 error rate per datapoint is a considerable improvement due to the low accuracy of the theoretical benchmark (0.44 error rate per datapoint for Duffie approach) but it would be very interesting to investigate whether there are areas where the CGP and GPR models are struggling or have an average similar behaviour for the whole dataset.

Figure 5.6 presents two occasions where both models have a poor accuracy in CDS price prediction. As presented, there are days when the CGP and GPR approaches predict a very low value for the CDS price while the actual market price of CDS is equal to 0. By checking the original data it was learned that the bond price of these days had also been 0. This is a different case compared to the non-availability of data that was discussed in Section 4.1. The other occasion is when the actual CDS price exceeds the boundary of 4 or 3, on some days, both the CGP and GPR models struggled and under-predicted the

Table 5.7: GPR Vs. CGP (experiment result). The first row presents the different approaches while the first column refers to different dataset folders as well as overall result mean, standard deviation, and datapoint error. The fitness result of different approaches were compared. For instance, the Duffie approach demonstrated a fitness number of 4482.1 (e.g. row 2, column 2) for folder number 1, while the CGP approach presented the fitness rate of 6921.6 (row2, column3).

	Duffie	GPR	CGP	Winner
Fold 1 fitness	4482.1	6921.6	6910.9	GPR
Fold 2 fitness	4488.4	6878.5	6886.1	CGP
Fold 3 fitness	4480.4	6868.9	6907.6	CGP
Fold 4 fitness	4481.3	6910.8	6906.4	GPR
Fold 5 fitness	4464.7	6899.4	6905.8	CGP
Mean	4479.38	6895.84	6903.36	
STD	8.789	21.946	9.848	
Datapoint Err	0.44008	0.13802	0.13708	

price of CDS.

This situation was probably due to the companys bankruptcy. In the CDS market literature (see Section 2.1), the price of a CDS is an indicator of a company financial health and the higher the price indicates the higher risk of default. When a company defaults or stops paying interest (bond yield) on a bond, or does not re-pay the principal at maturity, the bond defaults and consequently the bond yields and therefore the CDS price are equal to 0. The experts in the field were consulted on this point and they believed that the bankruptcy of the company can explain the situation⁷.

The CGP and GPR models did not manage to capture the sharpest spikes of CDS prices as extreme events are hard to predict. The common issue is that there are few extreme events in the data, therefore the model does not have enough samples to get trained.

Despite the similar behaviour of both suggested approaches, each has its own advantages and disadvantages. The computational time is a considerable issue to be discussed. The GPR approach is a relatively fast approach. The GPR model deals with the training set of 32000 data points and estimates the value of the test set (8000 data points) within minutes. In contrast the CGP model takes between 5 and 7 days (variation comes from the choice of initial random populations) for the same task. Therefore, the GPR approach is a favourable candidate for online situations when it is preferable to have a fast reliable estimation. However, GPR is a black box and therefore it is not possible to get a hint of what the predicted derived equation might look like while the CGP solution (e.g. chromosomes) can be translated to mathematical formulas or diagrams and thus enables more analysis. The CGP model is a favourable approach when the possibilities can be stored and traders can directly use them within a matter of seconds while having the opportunity to visualise and analyse the solutions, if required.

⁷The CGP and GPR models did not manage to capture the sharpest spikes of CDS prices as extreme events are hard to predict. The common issue is that there are few extreme events in the data, therefore the model does not have enough samples to get trained.

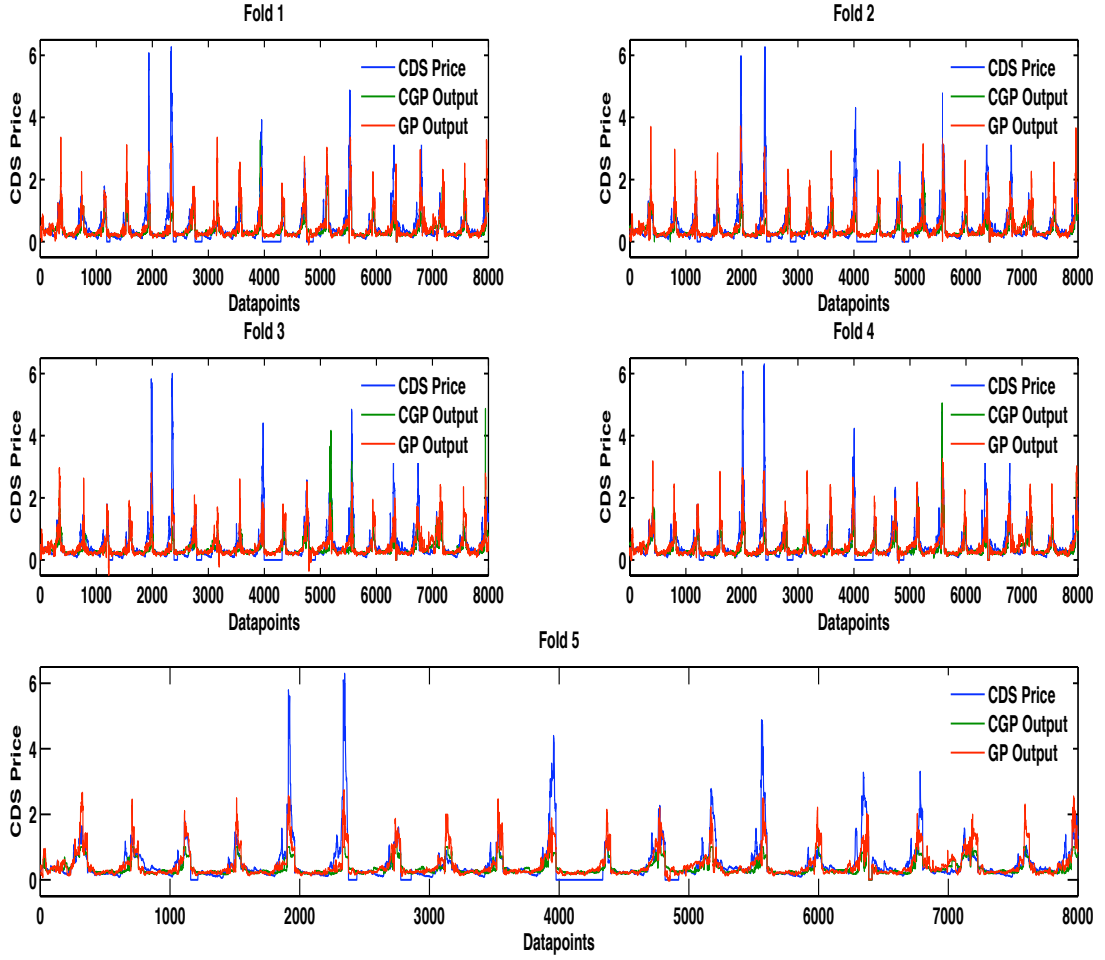


Figure 5.6: 5-fold Cross-Validation Result

The second objective was pursued by reducing the number of nodes from 50 to 40 to check whether it was possible to evolve a smaller solution while achieving similar accuracy as well as investigating the CGP behaviour. Figure 5.7 exhibits the results. As presented the CGP fitness reduced from 6905.8 to 6564.1 units. It seems that the CGP model struggles to achieve a better fitness by ignoring the price jumps. The CGP models demonstrated a similarity to previous experiments by its incapability in estimating the CDS price in a bankruptcy situation but it now also struggled in general for any kind of price fluctuation.

The best CGP solutions are visualised for 40 nodes and 50 nodes. Equation 5.10 exhibits the best solution when the number of nodes is set to 40. As presented, CGP ignored the interest rate and BP_{bid} price in its solution and consequently struggled to deal with price jumps. This is evidential as the interest rate has a direct impact on a company financial situation and CDS price.

$$CDS_{spread} = \frac{1}{BP_{ask}} \quad (5.10)$$

As presented in Figure 5.7 the solution successfully estimated the CDS price while it struggled with the bankruptcy situation. Figure 5.8 illustrated the best CGP solution when the number of nodes is set to

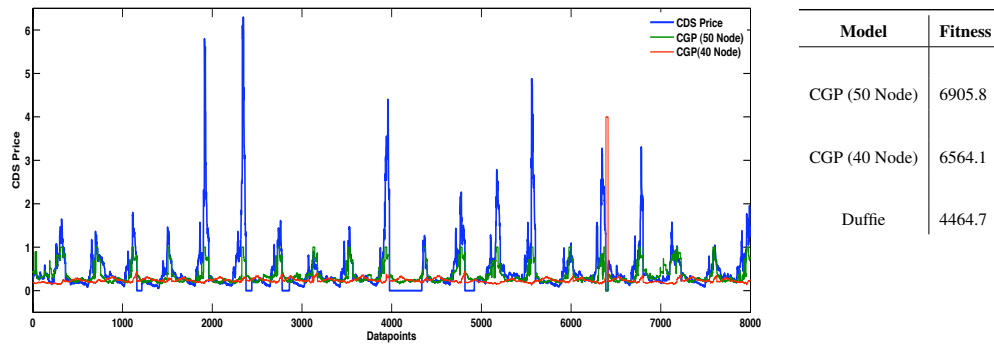


Figure 5.7: Node Comparison

50.

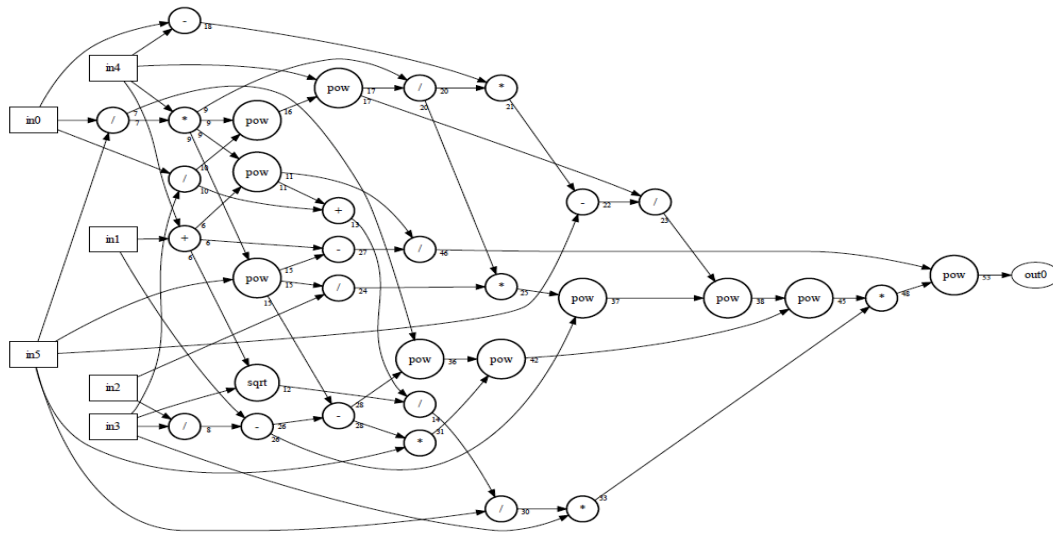


Figure 5.8: CGP function result for 50 nodes

It is argued that the inability of CGP to deal with a bankruptcy situation is not a CGP weakness because such situations are rare, the fluctuations are very unpredictable (big jumps and bankruptcy can happen in a few days) thus CGP cannot be trained properly.

5.4 Conclusions

CDS pricing is highly significant, not just for finance, but for the world economy. This is one of the first ever investigations into the CDS market using machine learning. In this work Cartesian Genetic Programming and Gaussian Process Regression were used to derive new relationships between variables in order to produce a dramatically more accurate model for CDS pricing compared to the standard Duffie approach. It was shown that CGP can completely outperform the standard pricing model, and some analysis is provided of the CGP solution, as well as the ability of CGP to cope with the financial data in comparison to its other competitors.

It was demonstrated that CGP is effective as a bio-inspired evolutionary method for a complex real-world financial problem. The data included the highly turbulent behaviour of the markets in the last

four years, with no loss of accuracy - a significant improvement over the Duffi method which showed a serious fall in accuracy. It was also demonstrated that the CGP parameters are sensitive and showed that CGP was able to provide more consistent results using fewer attributes. In addition, it was demonstrated that in the CGP setting variables like the number of nodes can strongly impact the CGP performance. Moreover, the strength of GPR approach was presented for dealing with a noisy dataset and its capability for distinguishing between related and unrelated data in the environment. These results show that the CGP and GPR approach behave similarly, however the advantage of using CGP over GPR in this case is that it is possible to extract the equation out of the CGP chromosome while the GPR is a black box from which it is not possible to extract a mathematical equation.

Although this may be the first use of GPR in finance, the results are highly significant and revealing. This suggests that other bio-inspired methods designed for noisy, unpredictable and unknown data may also be able to illuminate some of the hitherto murky waters of financial trading. It is anticipated that with tools such as these, future financial crises may be less likely to occur. The CGP technique will be made available for future traders and they will decide which solution should be employed for each specific situation.

The next chapter studies the third challenge called CDS Market Experimental Model where we explain our approach, discuss our result, and argue our position.

Chapter 6

CDS Market Experimental Model

The 2007-2010 financial meltdown illustrated the inefficiency and serious limitations of existing macroeconomic models for identifying problems in complex economic systems. As discussed in Chapter 1, traditional microeconomic models also fail to comprehensively describe market components and market behaviour. Consequently, the market is at risk and financial catastrophes are often not predicted in a timely fashion, but only after they have happened.

In the recent financial crisis, it was seen how a financial product such as a Credit Default Swap (CDS) contract can play a central role in a market crash. The CDS contract and market received the main blame for the 2007-2010 financial meltdown. This financial crisis highlighted the lack of experimental tools to analyse different market participants, market elements and market behaviour in various conditions and situations. Such a tool could increase the overall market understanding and transparency, and could help to avoid the entrance of risky products into the market.

This chapter is focused on the third and final challenge, the lack of practical CDS Market Experimental Model. We introduce our Agent Based (AB) approach for simulating CDS market. Our primary objectives are to design a CDS experimental tool with a focus on simulating market participants, market events and the negotiation process of the participants, to demonstrate how the AB model can help to analyse different elements of CDS market and study different market scenarios. The motivation of this work is to identify and tackle the steps needed to create a general purpose model for CDS trading. This may be used to investigate which factors result in equilibria¹ of market forces (e.g., to determine CDS pricing), however it makes no assumptions of equilibria and thus could be used to investigate a wide spectrum of possible markets including those with unstable or chaotic behaviours, depending on the parameters of the model.

This chapter consists of seven sections. We start with discussing our motivation (Section 6.1). Then, Section 6.4, describes our objectives, simulation process and assumptions, and experimental design. This is followed by three experimental sections (Section 6.5: Experiment 1, Section 6.6: Experiment 2, and Section 6.7: Experiment 3). Each experiment includes the explanation of the settings, data, and results. We evaluate our results by means of comparative evaluation and discussion in Section 6.8. Conclusions

¹Following are some significant works on using computational methods to approximate subgame equilibria: [Gosling et al., 2007], [Jin et al., 2009], and [Jin and Tsang, 2011].

are made in Section 6.9.

6.1 Agent Based Computational Economic: Motivation

There are multiple approaches in economics to the study of markets and market stability. One of the predominant approaches is to abstract certain properties of real world markets and to study equilibria of market forces in these systems. Equilibria are stable points in a system where different forces balance each other. For example, competitive equilibria are configurations in which the supply of some resource equals the demand. In many abstractions such a configuration is stable and configurations which deviate from it are forced toward the equilibria, e.g. if the supply is higher than the demand then there is pressure on the supply side to adjust to the demand and to not overproduce. Another important sort of equilibrium is called a Nash equilibrium. Within the game theory field, the Nash equilibrium is a solution concept involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy unilaterally [Osborne and Rubinstein, 1994]. In other words, a group of players are in Nash equilibrium if each one is making the best decision that he or she can, taking into account the decisions of the others.

Equilibria are of importance due to their properties of stability and the fact that, given certain assumptions, economic systems are forced into equilibria through properties of the system. Especially the property of stability is important in crisis times, i.e. given a system in an equilibrium state, how much force it will take to push the system out of this stable configuration and into potential volatile behaviour.

The study of equilibria is worthwhile and can provide insight into a system, though, a criticism of the approach is the abstraction and simplification that is needed to be able to study equilibria. Even properties like the existence of an equilibrium are difficult to verify in all but the simplest systems. Another point of criticism is the behaviour of the different agents in the system. Usually, it is assumed in these abstract models that these agents behave rationally and optimise their own profit. Yet, in the real world this is hardly true and there are many factors, like psychological factors, that have a significant impact on the behaviour of agents.

There are alternatives to this equilibrium approach which incorporate the human behaviour component in exchange for a fully rigorous and mathematical treatment of the systems. A typical approach is to build a well-controlled environment in which human participants interact and compete for resources. The experimenter then varies certain variables in the design to study the effect of these variables on the behaviour of the participants. This approach allows a far better insight into the psychological side of decision making under the assumption that the participants will most likely behave far from optimally as they are usually not trained in economical decision making. Nevertheless, the approach complements the equilibrium approach which takes a stance at the other extreme.

There are further problems with a psychologically-based approach, though. One of the most critical problems is that human participants tend to show remarkably inconsistent behaviour – compared to other participants and even if a single participant's behaviour is observed at different times. It is also nearly impossible to understand the driving forces that motivate participants to act in certain ways. Another difficulty with this approach is that experiments with human participants are time consuming, potentially

costly, and it is nearly impossible to study the interaction of more than a few dozen parties.

Agent based modelling is an extension of these experimental ideas which addresses the difficulties with human subjects. The fundamental approach is the same: construct an economical system, introduce different parties into the system and study the system behaviour while varying parameters of the system. The crucial difference is that the parties are now not human subjects but designed artificial agents. These agents have well defined goals and behaviour patterns that can be manipulated just like the system itself. This way there is no second guessing needed about what goal, if there is one at all, a participant pursues. Furthermore, experimentation is cheap as it is just computation time that is needed and simulating systems with thousands or millions of parties is possible.

We do not suggest that agent based systems should replace the equilibrium approach or experiments with humans, but, agent based systems are a natural companion for these two approaches which, in an optimal world, embed realistic human behaviour in the agent models and study, for example, equilibrium stability in sub-optimal and more realistic behaviour.

6.2 Experimental Objectives

The overall objective in creating the AB CDS market is to observe whether Agent-Based Modeling (ABM) is a reliable tool for policy makers and whether it can help find precipitating factors of future negative events. In this chapter we evaluate our model by focussing on three categories of experiments. Each category focuses on some specific setting of our model. We pursued two objectives in our experiments:

1. **Monitoring traders' behaviour under different strategies/policies.** We designed three categories of experiments called *conservative and non conservative*, *naive and smart*, and *risky strategy and reduced-risk strategy* by setting different features of our model. As discussed in Section 6.4.2, these policies affect the way that a trader performs its business.
2. **Checking the effect of different financial deals on the business.** It is of interest to observe how different financial deals (e.g. lending money, buying CDS protection or selling CDS protection) affect the traders' business.

6.3 Simulation Process and Assumptions

Figure 6.1 illustrates the agent-based CDS market process. The process starts from day 15. This is to provide 10 days historical information for the market. Since traders and risky companies might default due to our market events, the validity of the CDS market is checked at the beginning of each trading day. It is assumed that the market is valid if there are k_c number of risky companies and k_t number of traders who are active (not defaulted) in the market. k_c and k_t are constant integer parameters and there are set at the beginning of the simulation by the experimenter. Next the credit rating process is run to provide the traders financial information. Assuming that each trading month has 30 trading days, this process is run at the first day of each month. The market event function is called at this point in the simulation, so

the traders who have less than 10% of their initial asset or those with credit rate of “F” default. The risky companies might also default at this stage based on the reduced-form approach.

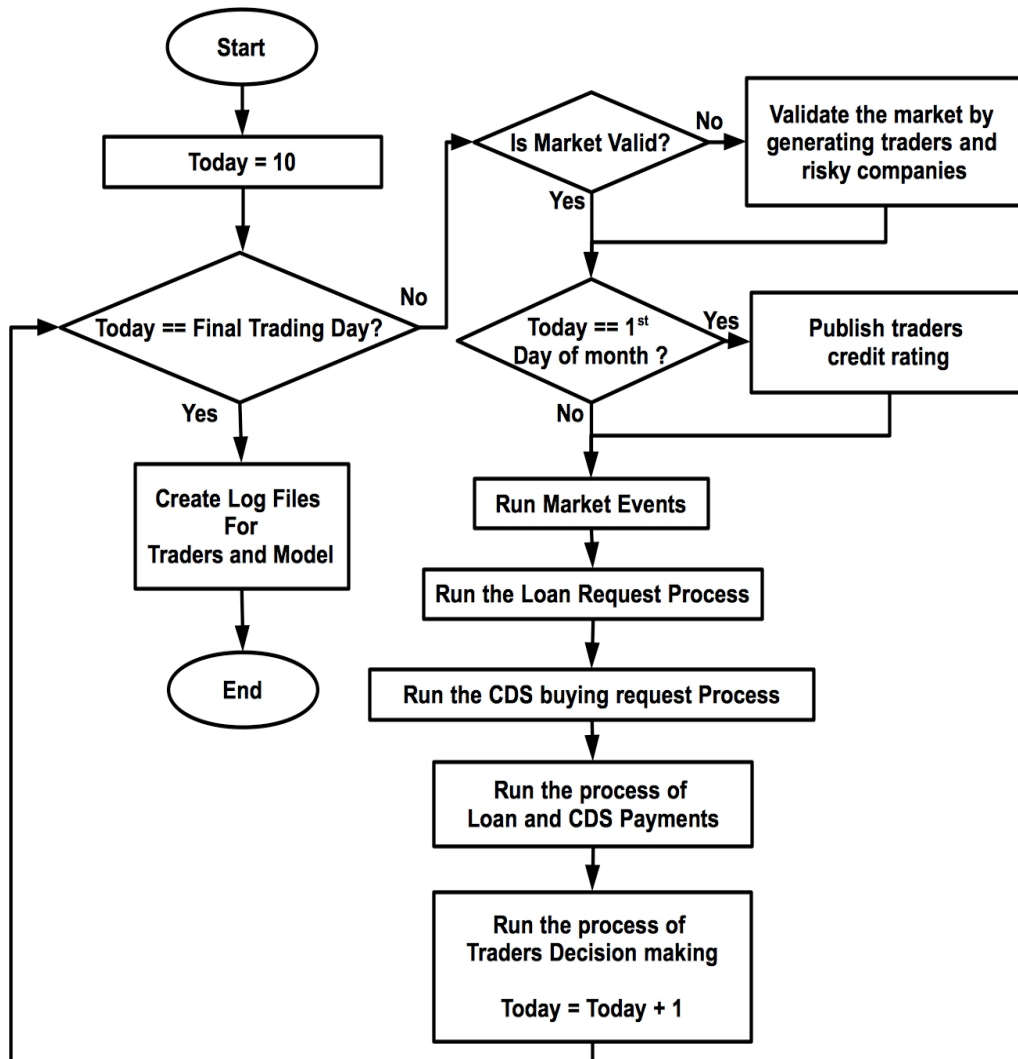


Figure 6.1: Overall Simulation Process

The risky companies make their loan request with probability of 50%. It is assumed that each company can apply for only one loan; therefore those companies which obtained a loan are excluded from this process. However, submitting a loan at this stage does not guarantee the approval of it. The requests remain pending until the end of the trading day when traders review the offers and make their decisions.

These decisions are followed by the process of buying CDS contracts. At this step, traders who own a loan can send a request to another for buying a CDS contract to protect its investment. Traders might use different strategies (e.g. Simple Moving Average (SMA)) to decide whether they should buy a CDS contract for a specific loan or not. It is assumed that only traders who own a loan can buy a CDS contract. Like the loan deal, submitting a request to buy a CDS contract does not guarantee the approve of it and this request might be rejected.

After submitting all the loans and buying their CDS contracts, traders update their asset information by running the payments function. Each trader reviews its own loan and CDS (only buying) lists and check whether they should receive or make any payments or not. The loan deals payments are made every 30 days while the CDS payments are made every 120 days. Traders go through their decision folder after the payment process. This is the final task of traders when they assess each offer value and decide whether they should make a particular deal or reject it. The outcome of each trade is completely dependent on its characteristic such as current asset value and trading strategy. This process is repeated for each trading day until the final trading day when the log information is created and the simulation is finished.

6.4 Experimental Design

This section describes the design process of an agent-based CDS market. Modelling all the details of a real CDS market at once can quickly lead to a complex simulation where it is difficult to determine which causal factors cause what effect. In this work, only on one part of CDS market is focused on where the CDS buyer owns the underlying asset and, therefore, there is no need to simulate the speculation sub-market.² This CDS experimental model illustrates CDS market participants, participant interactions, individual trading strategies and market events. There are two types of agents in the model: interactive agents and non-interactive agents. The non-interactive agents (e.g. the regulator and the credit rating agency) only provide information for the market. This information is provided on a central *blackboard* that is accessible to all agents. The interactive agents (e.g. traders and risky companies) are those who communicate and make business deals with each other. Figure 6.2 provides a conceptual visualisation of the CDS model participants.

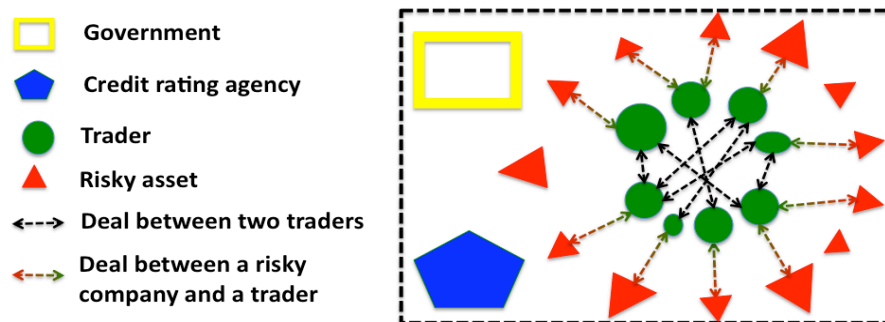


Figure 6.2: Agent-based Model of CDS Market. The agents size difference indicates the agents position in the economy. For instance, a bigger risky company refers to a riskier company and a bigger trader refers to a wealthier trader.

This section provides a comprehensive description of the experimental model, simulation process and experimental objectives. Firstly, the market participants are defined: risky companies, a regulator, a credit rating agency, and traders. The market participants are categorised into two types: Non Trader

²The CDS speculation sub market developed when the new parties became involved in the CDS market (e.g. buying a CDS contract) without owing the underlying asset. These new parties gamble on the possibility of a credit event of a specific asset.

Agents (Section 6.4.1) and Trader Agents (Section 6.4.2). To avoid unnecessary complexity, only those features and functionality's of market participants which are needed for the purpose of this work are modelled. The internal properties, actions and events of each agent kind are summerised in Table 6.1. Next the market events are explained: there are the structural approach and the reduced-form approach (6.4.3). Section 6.3 illustrates the simulation process. This is followed by a discussion of the experimental objectives in Section 6.2.

Table 6.1: Agents Key Features and Properties

Agent	Internal Properties	Actions	Events
Trader	Capital, Wealth Rank, Credit Rate, Flexibility Rate, Flexibility Type, Risk Preference, Pricing Technique, Trading Sterategy, Decision Folder	Give loan, Buy CDS, Sell CDS, Receive Loan Payment, Receive or Pay CDS payment	Structural Def.
Risky Company	Bond price	Get Loan, Pay Loan Payment	Reduced-form Def.
Credit Ratig Agency	- - -	Reports traders credit rating	- - -
Government	Interest rate and Treasury	Set interest rate and Collect money if necessary	- - -

6.4.1 Market Participants: Non Trader Agents

Risky company

A risky company is a company which is financially in trouble and needs to obtain a loan in order to maintain its business. The risky company is modelled based on the companys bond price. As explained in Chapter 4, there are two types of bond price: real world data and simulated data. The risky company tries to obtain a loan from a trader by submitting its request to a trader. This request includes the *amount of loan* , *start date*, *expiry date*, and *interest rate*. Although these features can be varied if needed, the amount of the loan is set to 10,000,000 which is typically the starting range for a CDS contract national amount (Section 2.1). A fixed interest rate of 25% is used for all loan deals. To apply a realistic interest rate a quotation for a 10,000,000 loan was requested from several banks but they did not respond because they require the applicants full business details which we could not supply. Therefore the interest rate that Barclays bank applies for its credit card user (interest on cash) was used. The risky company bond information is publicly available in the market; thus market participants can use this bond information as an indicator of the companys health.

Regulator

A regulator has two properties in our model: interest rate and treasury rate. The regulator provides daily an interest rate for the market. Other market participants may use this interest rate for different purposes such as pricing a CDS contract. A real word interest rate obtained from Bank of England was used which matched our real world bond prices time-stamp. The regulator also acts on behalf of other market participants if needed. For instance, if a trader defaults, the regulator collects its payments such as earned interest on loans and CDS contracts.

Credit Rating Agency

The credit rating agent is considered to be an independent company who observes the traders asset, As , and publishes the credit rating of traders regularly (e.g. every three months). Traders can use this information for assessing each other's financial situation. The process of rating includes classifying the traders based on their current asset relative to their initial capital amount and the maximum and minimum of the market capital allowance. Table 6.2 demonstrates the credit rating system of this model.

Table 6.2: Credit Rates Table

Wealth category	Sw	W	Aw	Nw
Asset status				
$80\%C \leq As \leq \text{Max}$	AA	A	A	B
$60\%C \leq As < 80\%C$	A	B	B	B
$40\%C \leq As < 60\%C$	B	B	C	C
$20\%C \leq As < 40\%C$	C	C	C	D
$10\%C \leq As < 20\%C$	D	D	D	D
$0 \leq As < 10\%C$	F	F	F	F

As denotes the current amount of the traders asset. C indicates the traders initial capital amount. The wealth category has four subcategories: Super Wealthy (Sw), Wealthy (W), Average wealthy (Aw) and Not wealthy (Nw). This refers to the position of traders capital relative to the maximum and minimum of the market capital allowance. The market credit ranks are indicated by AA, A, B, C, D, and F, where AA is the top rank and F is the default rank.

6.4.2 Market Participants: Trader Agents

A trader is a financial institution. It performs business in two ways: either lending money to a risky company or selling a CDS contract to another trader. A trader can also decide to get a protection for its existing loan contract in order to reduce the risk of losing money due to company default. Within this experimental model, a goal of a trader is to make profitable deals and avoid risky deals. A trader has a specific amount of capital C , a wealth rank, a credit rate, a flexibility rate f , flexibility type, a specific risk preference r , a pricing technique, a trading strategy, and a decision folder.

The trader initial capital amount, C , is set randomly at the beginning of the simulation based on the market capital allowance:

$$Market_{minCapital} < T_i C < Market_{maxCapital}.$$

Based on this capital amount, the trader's wealth rank and credit rate are assessed by the credit rating agent periodically (see Table 6.2).

The flexibility rate f is set at the beginning of the simulation by experimenter. The flexibility rate defines the traders willingness for decreasing or increasing the offered price. A trader has two types of flexibility: active or passive. Traders have to choose an active or a passive flexibility rate to control their negotiation rates and price range. A passive flexibility means a trader chooses its preference at the beginning of the simulation and it cannot change it after that. However, traders with an active flexibility factor are capable of modifying their preference during the simulation.

The risk preference r , demonstrates the trader's threshold for decision making. Traders rate and evaluate the available offers against a threshold. Those offers which are above the threshold, r , will be made while the remaining offers are rejected. The risk preference r is drawn from a Gaussian distribution at the beginning of the simulation:

$$r = N(0, 0.001)$$

Trader Business Deal

There are three kinds of business deal available to traders: a loan deal, buying a CDS contract and selling a CDS contract. A loan deal is made between a risky company and a trader. A CDS contract deal (buy/sell) is made between two traders. These deals are traders' financial tools to perform business. A trader makes money by receiving monthly or quarterly payments called premiums from a risky company or a CDS buyer. A trader has the opportunity to protect its money lending investment by buying CDS protections for its loans. In Section 2.1, it was explained that an investor can exit from a CDS contract before the contract expiration date. However, in this model, traders are obliged to the deal until the expiration of the contract or a default event to reduce the undesired complexity. Both deals involve some level of risk as a trader might lose its money if the risky company or the trader defaults (fails to pay the payments). In the money lending process, if a risky company becomes bankrupt then it cannot pay back the loan and the lender loses its money. In the CDS contract, a CDS buyer receives compensation from a CDS seller if the risky company defaults, therefore, the CDS buyer covers part of his or her loss and the protection seller might lose/gain money depending on the premium rate, recovery rate, and default date. The protection seller's gain or loss can be calculated by deducting the recovery payment from the sum of its received premiums. Table 6.3 summarises loan and CDS deals key features.

Table 6.3: Contracts Features

Contract Type	Features
Loan Contract	Borrower, Lender, Amount, Start Date, Length of Contract, Expiry Date, Interest Rate.
CDS Contract	Protection Buyer, Protection Seller, Risky Company, Amount, Start Date, Expiry Date, Recovery Rate, Offered Price.

A loan deal requires the submission of a loan request from a risky company to a specific trader. The loan request includes the amount of money and terms of the loan contract such as interest rate and length of the contract. Each trader reviews the contract terms and it agrees or disagrees to sign the contract based on its own asset limit, the risky company financial situation (bond price), and other available deals. By signing the contract the risky company starts paying its periodic fees regularly until the end of the contract to the trader. If the risky company defaults before the expiration of the contract, then the trader loses its remaining money.

In the process of a CDS deal, the protection buyer creates and submits a CDS contract to a seller. The contract offered by the buyer includes the underlying asset (risky company) information as well as the contract terms such as recovery rate and length of the contract and the offered price. The seller

reviews the contract terms and it calculates its own price and submits its offered price to the buyer. If they both ask for the same price then the deal is agreed, otherwise they negotiate on the price. Submitting the same price is a rare event as the buyer tries to pay less and the seller tries to pay more. This is included by introducing a profit factor, σ_p . Initially to ensure simplicity for the model, the Duffie and Hull approach is used [Hull and White, 2000] to calculate the CDS price through a simple relationship and the profit factor σ_p is added to it.

$$Buyer_{spread} = (B_y - Ir) - \sigma_p \quad (6.1)$$

and

$$Seller_{spread} = (B_y - Ir) + \sigma_p \quad (6.2)$$

where the B_y is the risky company bond yield price, Ir is the interest rate also called risk free bond price. The σ_p is defined as $\sigma_p = (B_y - Ir)\alpha$ where α is a random number within the trader flexibility range. (Chapter 7 describes the integration of the CGP-generated CDS prices with the model.)

Trader Negotiation Process

If the protection buyer offers a price which is greater than or equal to the desired protection seller price, there is an overlap and the agreed price is $(Buyer_{spread} + Seller_{spread})/2$. In case the protection buyer offered price is less than protection seller offered price, the traders enter the negotiation process (see Figure 6.3). The negotiation process is a limited process of $N = 10$ rounds of bargaining. N is set by the experimenter at the beginning of the simulation. At any round of the negotiation process, if protection buyer meets the protection seller expectation then the deal is made and the agreed price is equal to $(Buyer_{spread} + Seller_{spread})/2$.

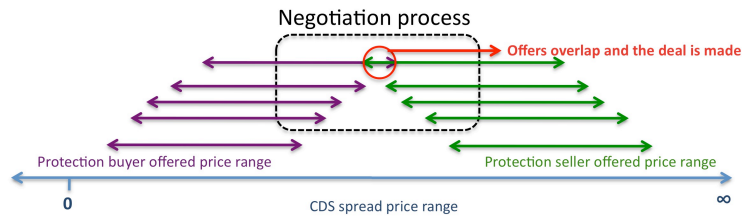


Figure 6.3: Price Negotiation Process

If the offered prices do not overlap at any of the N negotiation rounds then the deal cannot be made and traders exit the negotiation process.

The negotiation process is a heuristic process and it is based on flexibility rate". At each round of the negotiation process, the protection seller decreases its desired price and the protection buyer increases its desired price according to its flexibility rate. The deal price is referred to as an agreed price between the protection buyer and the protection seller after a process of negotiation.

Each trader has an error calculator tool for assessing how good the deal has been and what strategy should be used for the next trading period (e.g. changing flexibility rate). In this version of the model where real-world data is used, the CDS market price is considered as a measure of goodness of the trader

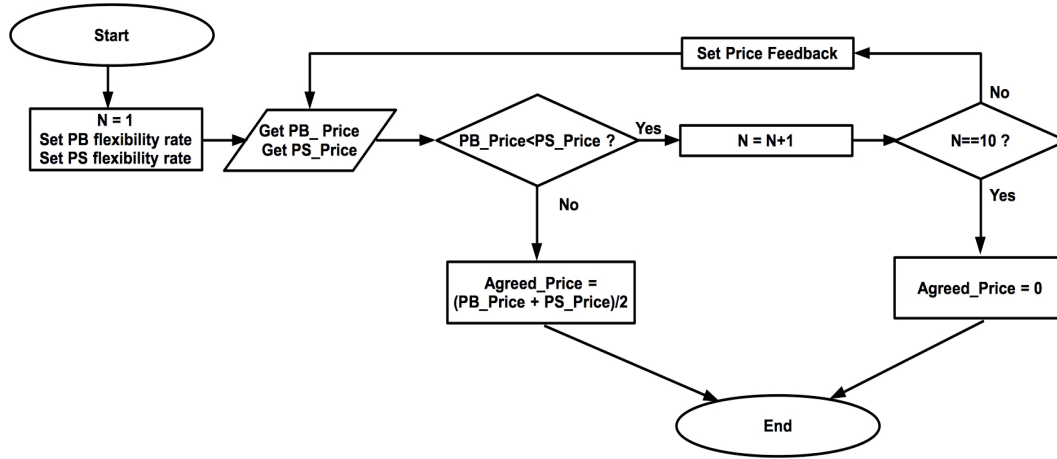


Figure 6.4: Price Negotiation Process Flowchart

activity. In the case of the protection buyer, a good deal is a deal which is priced below the CDS market price, ($PB_{Error} = CDS_{MarketValue} - Agreed\ Price$). Therefore, the protection buyer would like to have a positive error as a feedback on the trading transaction and the higher error means a better deal as he pays less for the contract. On the other hand, the protection seller considers it a deal a good deal if it is higher than the CDS market value ($PS_{Error} = Agreed\ Price - CDS_{MarketValue}$). The protection seller, would also like to have a higher error at the end of the trading transaction as the higher error means more profit for the protection seller. The error factor could act as a guide for traders for choosing its flexibility range for the next trading period.

Let us consider a situation where protection buyer offers the CDS spread of 5 and the protection seller offers CDS spread of 4.5, in this case the deal would be made at the first try since the protection seller expectation is met. However, if the protection buyer offers the CDS spread of 4.5 and the protection seller offers CDS spread of 5, no price overlap. In this case, traders start the negotiation process based on their flexibility rate. Assume that the protection buyer has a flexibility rate of 15% and protection seller has a flexibility rate of %10. At the first round of the negotiation process, the protection buyer offers the spread of 5.175 ($4.5 + 4.5 * 15\%$) and the protection seller offers the spread of 4.9 ($5 - 5 * 10\%$). Due to a generous offer from the protection buyer, the sellers expectation is met at this round and the deal is agreed. In case of no overlap at this round, traders would repeat the same process for N times. When a deal is agreed it means that both dealers are happy to consider this deal, however, the final decision of each trader is not only based on a single offer. A trader goes through all the available business deals and then decides which deals he or she would like to make. The traders preference is based on the type of trader strategy that he or she has.

Trader Strategies

There are two types of strategies available to traders called: *Basic Strategies* and *Advanced Strategies*. The basic strategies category assumes that a trader has no financial information about the other parties. The advanced strategies category assumes that the trader obtains other parties' financial information before making its decision. The details of each category is provided in the next sections.

A) Basic Strategies. Within the category of basic strategies, two kind of policies are available in the model: *Naive* and *Smart*³. Each trader randomly decides which strategy to take at the beginning of the simulation and will keep that strategy till the end of simulation period. Each trader has a decision folder, in which it keeps all the candidate decisions that need to be considered. The decision folder is reviewed at the end of each trading day and is reset before the next day. A naive trader goes through the decision folder and makes the decisions based on a First-come, first-served (FCFS) approach. In contrast, a smart trader prioritises its folder in three stages. The first is to give the same priority to all decisions: $P(di) = 1/n$ where $P(di)$ is the priority of the decision number i and n is the number of current decisions in the folder. The second step is to change the priority of each decision based on the nature of the decision. This is done by introducing a scaling factor β , $\beta = 0.1D_t$, to the model. Where D_t is the decision type and set to 0 for a loan deal, 1 for selling a CDS contract and 2 for buying a CDS contract. The final priority of decision number i equals to $Pr_{di} = P(di) + \beta$. Therefore, β increases the priority of buying a CDS contract by 0.2, selling a CDS contract by 0.1 and lending money by 0. Finally, the smart trader sorts its decision folder and agrees on m/n of decisions, where m is a randomly distributed number. Figure 6.5 illustrates the process of decision making in the AB CDS market.

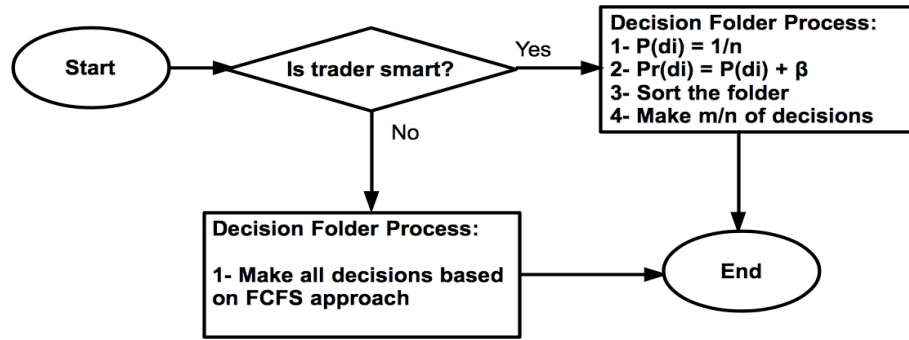


Figure 6.5: Decision Making Process Flowchart

B) Advanced Strategies. Traders are a central part of this model and for the model to be valid as a predictive tool it needs to resample their behaviour as effectively as possible. While it is difficult to obtain data from the trading behaviour of real traders, it is clear that they try to maximise some performance measure if they act rationally. The following assumption will, therefore, be made:

Real traders act rationally.

This assumption allows a link to be made between the agent behaviour and the behaviour of real traders: It is assumed that there is a unique strategy that optimises the performance measure then if both the trader and the artificial agent act optimally with respect to the same performance measure then the agent behaves like the trader. If it is assumed that traders are inherently selfish, an optimal strategy for a trader is the strategy that maximises the profit for that trader, regardless of the effect of the strategy on other traders. If there is more than one optimal strategy, then it seems safe to assume that real traders exploit

³This strategy is called Population Based Incremental Learning (PBIL). See [Alexandrova-Kabadjova et al., 2011] for an application of PBIL in agent-based economics.

the different solutions and trader behaviour can be represented by finding the different solutions.

There is some degree of freedom in the choice of the performance measure, but a sensible metric would be the trader's profit. In reality, it is difficult to find a strategy that optimises profit (this is a very active research area on its own as a strategy that results in more profit means potentially a large gain).

Deriving profitable strategies in complex settings is, however, a very challenging task and beyond the scope of this thesis. Instead, we introduce a layer of abstraction to the behaviour of a real trader that like the optimisation in the more complex setup, needs to be studied in future works (see also the discussion below). For the sake of simplicity and to enable the behaviour of the model to be more comprehensible, we make the following assumptions:

1. **Decision Time Interval:** agents decide about loans and CDS contracts on a daily basis instead of planing ahead for weeks or months.
2. **Decision Prioritization:** the agent rates deals and accepts all deals that are rated above a fixed threshold.

Rating of Offers and Decision Making: As explained above, there are two categories of offers: loan deals and CDS deals. In the case of a loan deal, risky companies approach traders and ask for loans. In the case of CDS deals, traders buy or sell CDS contracts from other traders to perform their business.

Within the advanced strategies, we rate offers based on credibility of the parties involved (e.g. companies and traders) in the deal. For a loan deal, traders use the bond yield information of the risky company as a measure of the quality of the company. The traders use the *SMA* technique to check the current company trend based on the company n days historical price. The moving average technique is one the most popular technical indicators which are used to gauge the direction of the current trend. The moving average is calculated by averaging a number of past data points. It allows traders to look at smoothed data rather than focusing on the day-to-day price fluctuations that are inherent in all financial markets [Murphy, 2006]. Thus the today's *SMA* (SMV_{Today}) for a n -day sample of bond price (BP) is the mean of the previous n days' bond prices. If those prices are $BP_M, BP_{M-1}, \dots, BP_{M-(n-1)}$ then the formula is

$$SMV_{Today} = SMV_{Yesterday} - \frac{BP_{M-n}}{n} + \frac{BP_M}{n}$$

Where

$$SMV_{Yesterday} = \frac{\sum_{i=M-(n-1)}^{M-1} BP_i}{n}$$

We then define the company indicator as

$$CompanyIndicator = SMV_{Today} - SMV_{Yesterday}$$

In the case of CDS deals, there are two strategies available: the risky strategy and the reduced-risk strategy. In the risky strategy the trader rates a CDS deal only based on a reference entity (risky company). The risky strategy does not consider the risk of the counterparty involved in the deal. In contrast to the risky strategy, the reduced-risk strategy consider both reference entity risk and counterparty risk.

The traders use the credit rate of other traders to estimate the counterparty risk. The credit rating system is explained in Section 6.4.1. The rating gives us a number between 0 and 5 where 0 means an excellent trader and 5 indicate a defaulted trader.

The ratings need to be combined and evaluated against a threshold. In the reduced-risk strategy, we combine them by normalising the trader rating to be in the magnitude of the company trend and by then taking the minimum of the two: the minimum means that the trader considers the smaller rates among the opponent trader rate and the risky company rate. The threshold is picked randomly for each trader around 0 (we use a normal distribution with the same normaliser of the trader rating). The motivation is here that the threshold encodes the risk preference of the trader and picking it randomly means that different traders have different risk preferences. More formally:

- Risky strategy is defined as

$$ac \geq r \Rightarrow \text{deal},$$

$$\text{else no deal.}$$

Here, a is risky company trend indicator, r is the risk preference of the trader which needs to decide about the deal and c is 1 if the deal is a loan request or a request to buy a CDS insurance from another trader. $c = -1$ if the trader itself is thinking about insuring a company deal. For instance, when the company trend indicator shows a negative movement in the bond yield price then the trader tries to buy CDS protection for that risky company to reduce its risk.

- Reduced-risk strategy is defined as

$$\min\{ac, sp\} \geq r \Rightarrow \text{deal},$$

$$\text{else no deal.}$$

Here, s is the scaling which was set to 0.01 in these experiments and p is 2.5 minus the rating of the opponent trader, i.e. trader rating 3 and 4 is around the threshold where it depends on the risk preference if a deal will be made.

Discussion of Assumptions The goal of this research is not to match trader behaviour and market behaviour exactly, but to provide a basic model and a plan of attack for how the approach can be refined successively to become more realistic. The plan of attack for our artificial traders is here to find more refined strategies that are better in maximising profits and through that to better simulate real traders. This reasoning is based on our assumptions, which on their own are an interesting object of study. The assumptions provide a way to reproduce trader behaviour without having the need to get real world data, which is very difficult to obtain since traders are not happy to publish their successful trading strategies. In the course of this research numerous discussions were held with financial trading experts and the experience gained from these conversations informs this approach.

6.4.3 Market Events

The simulated market events are of two types: the structural approach and the reduced form approach. In 1974, Merton proposed that the evolution of the company asset follows a diffusion process. In other

words the defaults occur when the value of the asset becomes lower than the debt. A firm never defaults by surprise as a diffusion process is continuous and a sudden drop in the company asset value is impossible [Navneet et al., 2005]. We model the structural approach by assuming that the firm would default if the asset value, As , falls below a certain default boundary X , where X is defined as a portion of the initial asset value, C .

In contrast to the structural approach, the reduced-form approach assumes that there is no relation between the firm value and default. This approach was proposed by Duffie in 1998 and the idea behind this approach is that the credit event can be (approximately) modelled as a Poisson process, with hazard rate (or intensity rate) h depending on the length of the time interval. This implies that the expected number of credit events between time 0 and t is ht . The probability for k events in the interval 0 to t is

$$\frac{\exp(-ht)(ht)^k}{k!} \quad (6.3)$$

The reduced form approach was developed for a single company and does not allow for multiple defaults per time step. Since this simulation includes hundreds of companies single defaults per time step are not a viable option and the reduced form approach was extended to allow for multiple defaults per time step while incorporating the direct or indirect impact of the interest rate on the companies. We did so by depending the number of defaults on the change in interest rate. So like in the standard reduced form approach no companies will default if there is no market event, however, if there is a market event then

$$\frac{n_{Ac}Ir_H}{\mathcal{N}}$$

many companies will default at time t where n_{Ac} is the number of active companies, Ir_H the interest rate change from time $t - 1$ to time t and \mathcal{N} a normalising factor.

In our CDS AB model, the risky companies default only through reduced form events to incorporate the unknown financial market event while traders default through the structural approach to reduce the unknown financial market event thus enabling the detail investigation of traders activities.

The following sections explain the details of the three experiments and the associated results. Each experiment is focused on specific aspect of the model.

6.5 Experiment 1: Conservative and non Conservative Category

6.5.1 Experimental Datasets, Settings

In Experiment 1 category, the trader's CDS pricing capabilities were the focus based on their conservative and non conservative behaviour. The simulation was begun with two traders who are forced to price a CDS contract every day. Traders are not going through the trading process at this stage since only the price and flexibility rate are of interest. 20 experiments were run and each experiment is focused on one CDS contract (there are 20 real data companies, see Section 4.1). This experiment covers 400 trading periods. Each trading period can be considered as a trading day since the CDS spread price is available daily.

The flexibility status can be set to “on” or “off” (to active or passive) at the beginning of each experiment. The trader set its flexibility rate also at the beginning of the simulation. A conservative trader has a passive flexibility rate which makes a trader capable of going through the negotiation process and change its spread offered price but he is not able to change his flexibility rate within the trading period to gain more profit. The trader with an active flexibility rate is considered as a non-conservative trader and he is able to change his flexibility rate within the trading period in order to change the rate of increasing or decreasing the offered price. Each trader set his maximum and minimum flexibility rate at the beginning of each experiment. The interest rate and risky company bond information (bond ask price and bid price) are available to both protection buyer and protection seller. The daily CDS market value becomes visible to both traders at the end of each trading day (when the deal is made between buyer and seller).

Two different types of experiments were performed to monitor the model dynamic based on the traders behaviour. These experiments are mainly focused on the heuristic behaviour of traders. Traders can employ a very basic level of heuristic learning by defining their own flexibility status and rates. In these experiments, traders have no information about the CDS market value at the time of trading and the CDS market value becomes visible to them when the deal is made only for the purpose of error feedback. Moreover, the traders have no access to historical information (memory-less).

Experiment A: Conservative Buyer vs. Conservative Seller. Experiment A, investigates how the conservative behaviour of traders affects their error rate and final agreed price. The error rate is the difference between the CDS market value and the agreed CDS price where the agreed CDS price is a price after a negotiation process between buyer and seller, see Section 6.4.2. There are three sets of experiments in this section. In set 1, the protection buyer and the protection seller have the same flexibility rate (Flexibility rate = 10, 50, and 90). In set 2, one trader’s flexibility rate is kept fixed at the lowest boundary (Flexibility rate = 10) and different flexibility rates are tried (Flexibility rate = 10, 50, and 90) for the other trader. The set 3 includes experiments where the traders have opposite flexibility rates: one is set to a high rate and the other one is set to a low rate. These experiments are based on real data and the CDS market value is used for calculating the traders error, called feedback error. The feedback error is the difference between the agreed price and market value. Only the error rate of the protection buyer is demonstrated as the error rate of the protection seller is $ErrorRate - 1$.

Experiment B: Non-Conservative Buyer vs. Non-Conservative Seller. Experiment B, monitors non-conservative trader behaviour. The non-conservative traders have the same pricing and error calculation techniques as conservative traders but they have an extra feature, flexibility status, for changing the flexibility rate within the trading period. This extra feature helps the trader to increase his performance by decreasing his flexibility rate if his error is negative. This feature helps the traders to act more reasonably based on their error feedback. There are two sets of experiments in this section. Experiment set 1 is focused on a situation where the protection buyer and protection seller have the same flexibility rate (Flexibility rate = 10, 50, and 90). In experiment set 2, the protection buyer flexibility rate is fixed at the low boundary of 10 while the protection seller flexibility rate varies (Flexibility rate = 10, 50, and 90).

6.5.2 Result

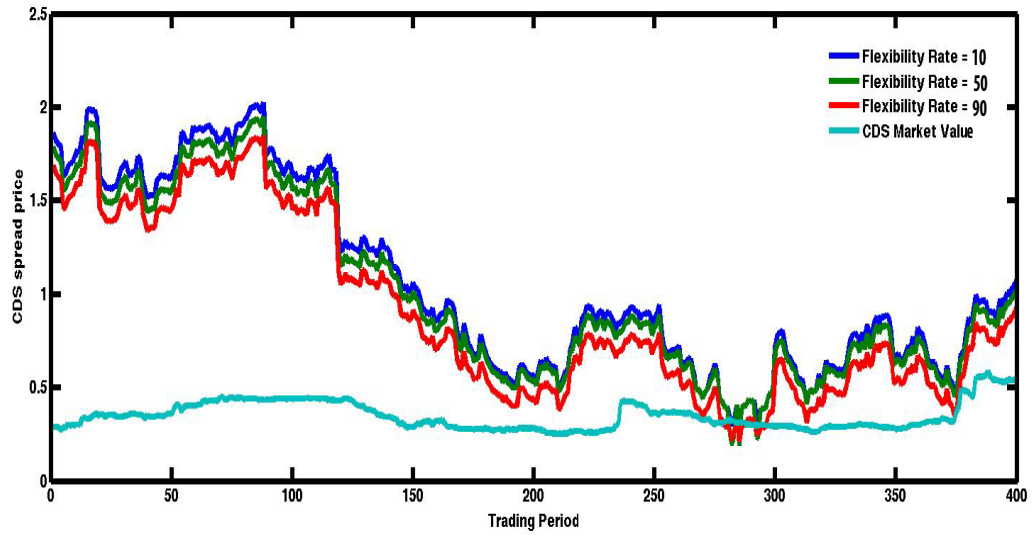
Result A

Figure 6.6 and Figure 6.7 illustrate the traders agreed price and feedback error for set 1, 2, and 3. The CDS prices are averaged over twenty experiments for each flexibility rate.

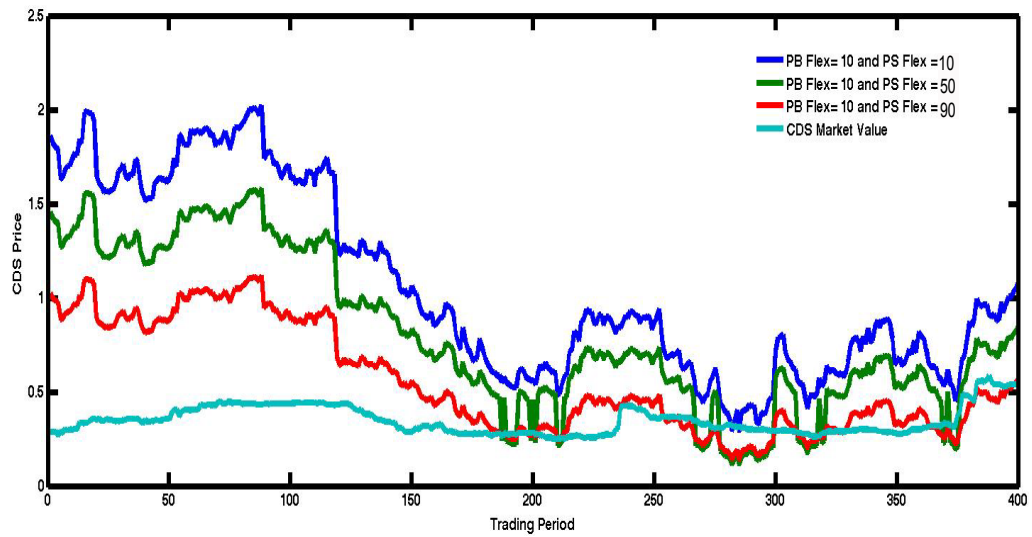
Results A.1: Conservative Buyer vs. Conservative Seller (Set 1). According to the result shown in Figure 6.6(a), when both traders have the same flexibility rate the lower flexibility rate leads to a higher CDS price (blue curve) and the higher flexibility rate leads to a cheaper price (red curve). However, the fixed and equal flexibility setting does not let the agreed price to be very different and the average difference between the agreed price when the flexibility rate is 10 and 90 is 0.2. The only reason that the agreed price gradually gets smaller is that the protection buyer and seller increase and decrease the price offer in order to make an overlap (see section 6.4.2 for the negotiation process in detail). Figure 6.7(a) demonstrates a higher error rate for traders with higher flexibility rate (red curve) and lower error rate for the traders with lower flexibility rate (blue curve). This setting is highly desirable for the protection seller since the contract is overpriced, thus the protection seller error is minimised. The result from experiment set 1 suggests that the flexibility rate is positively correlated with the error rate and negatively correlated with the CDS price.

Results A.2: Conservative Buyer vs. Conservative Seller (Set 2). The experiment set 2 replicates the same finding as experiment set 1, see Figure 6.6(b), showing that the higher flexibility rate leads to a cheaper CDS price (red curve) and the lower flexibility rate leads to a higher CDS price (blue curve). Like experiment set 1, our feedback error (see Figure 6.7(a) and Figure 6.7(b)) is showing a higher error rate (red curves) for a higher flexibility rate and a lower error rate (blue curves) for the lower flexibility rate. However, in contrast to the experiment set 1, the average difference between the agreed price when the flexibility rate is 10 and 90 is 0.6. Our results show that a protection buyer with a low flexibility rate has a good chance of making a good deal by choosing a protection seller with a high flexibility rate and vice versa.

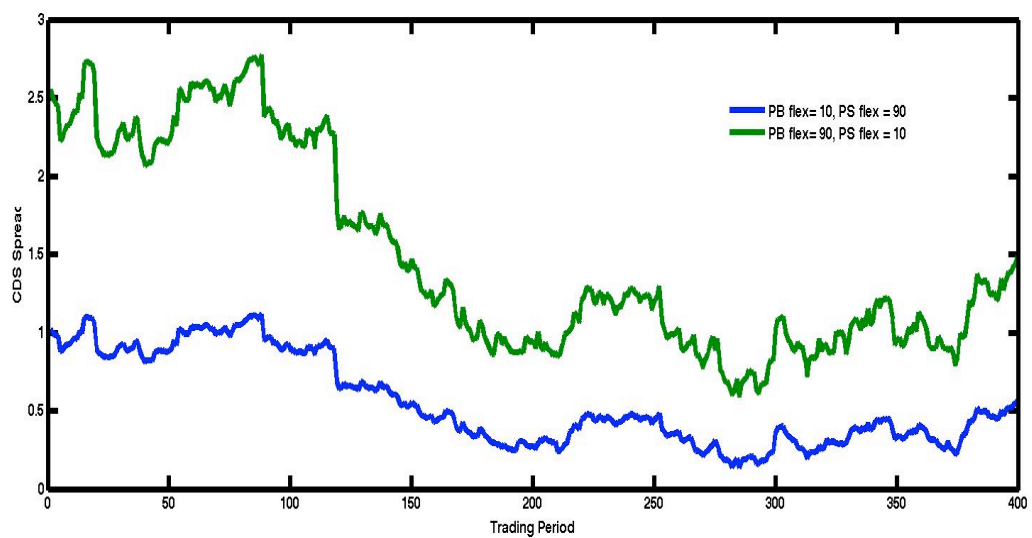
Results A.3: Conservative Buyer vs. Conservative Seller (Set 3). Experiment set 3 assesses the impact of a high difference in the traders flexibility rate. As illustrated in Figure 6.6(c), on average, there is a 1.2 difference between the two offered prices (green and blue curves) which is double the value of what was observed in the experimental set 2. The error Figures 6.7(b) and 6.7(c) demonstrate that even with almost no memory and no learning capability, there is still a possibility to make a profitable deal by choosing a stupid trader. As mentioned in Section 6.5.1, the traders do not have any learning capability and the flexibility rate is an internal trader characteristic. Therefore, the traders cannot specify which trader has a higher or lower flexibility rate in order to take advantage of the situation. Later on in this chapter, it will be shown how traders will assess each others behaviour in order to choose a better trader and make more money.



(a) Exp. A - Set 1: Protection buyer and protection seller have same flexibility rate of 10, 50, and 90.

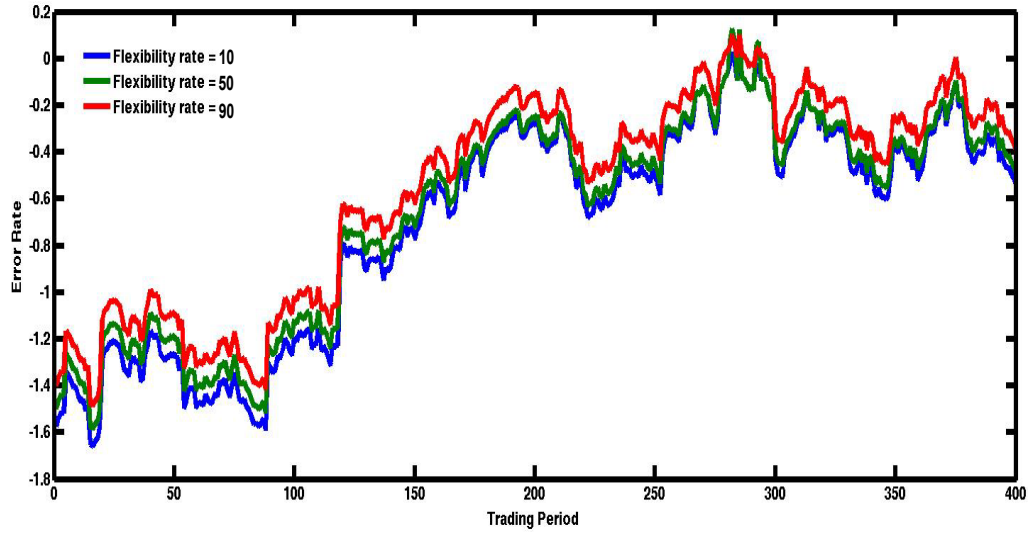


(b) Exp. A - Set 2: Protection buyer has a fixed flexibility rate of 10 while the protection seller has a flexibility rate of 10, 50, and 90.

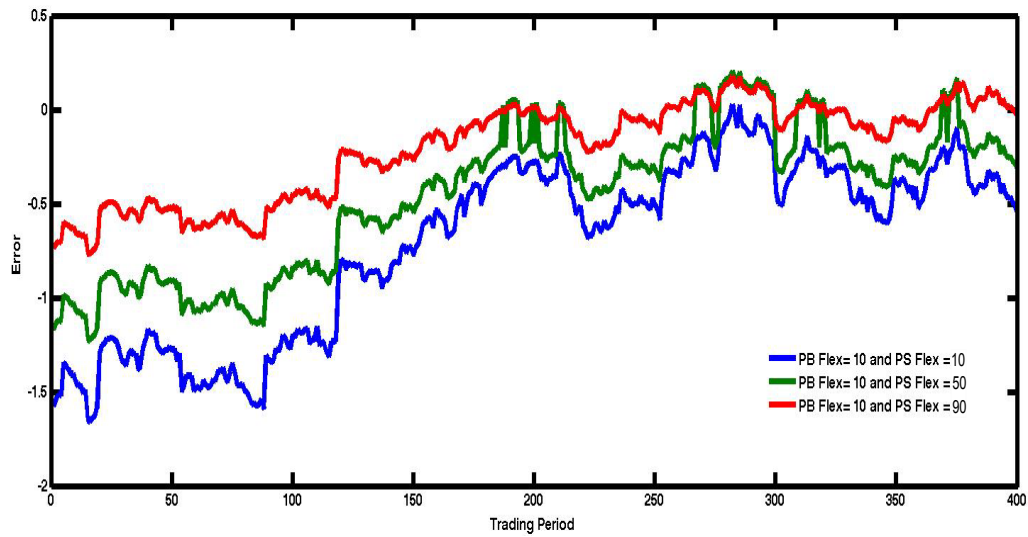


(c) Exp. A - Set 3: Protection buyer and Protection seller have opposite flexibility rate.

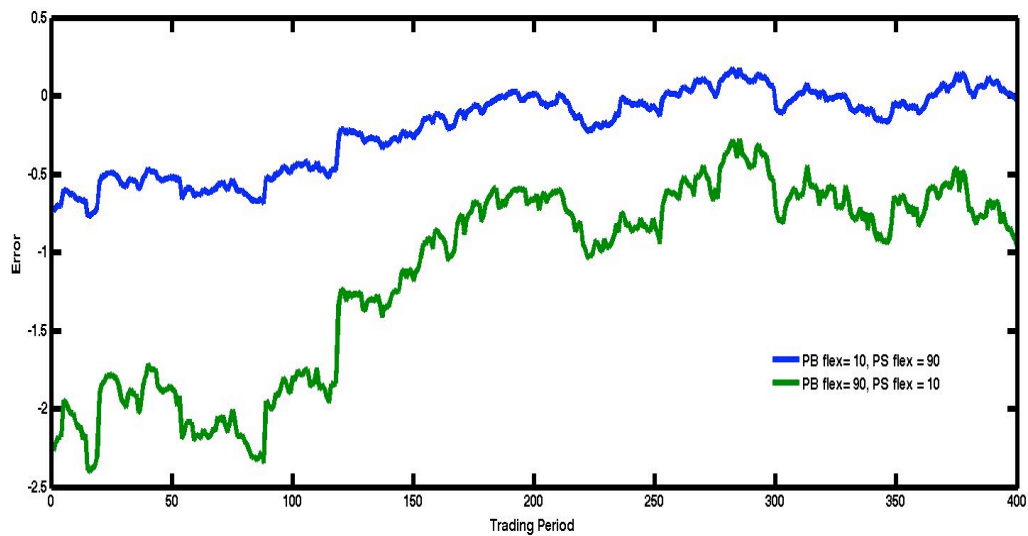
Figure 6.6: Conservative Buyer Vs. Conservative Seller: traders agreed prices according to different flexibility rate.



(a) Exp. A - Set 1: Protection buyer and protection seller have same flexibility rate of 10, 50, and 90.



(b) Exp. A - Set 2: Protection buyer has a fixed flexibility rate of 10 while the protection seller has flexibility rate of 10, 50, and 90.



(c) Exp. A - Set 3: Protection buyer and Protection seller have opposite flexibility rates.

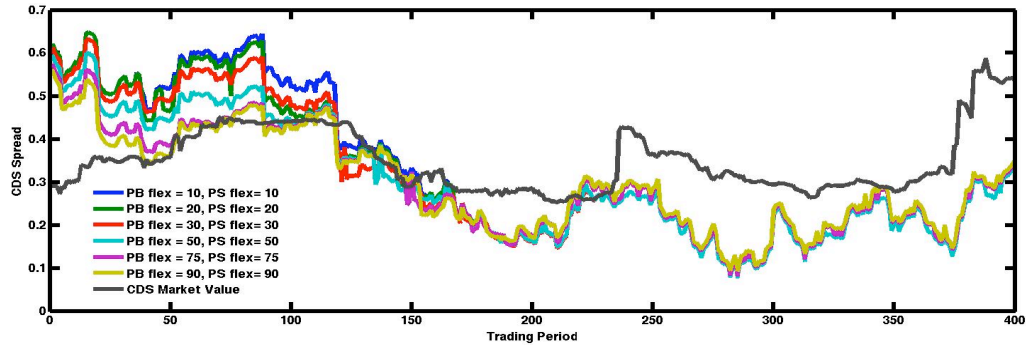
Figure 6.7: Conservative Buyer Vs. Conservative Seller: traders feedback error according to different flexibility rate.

Result B

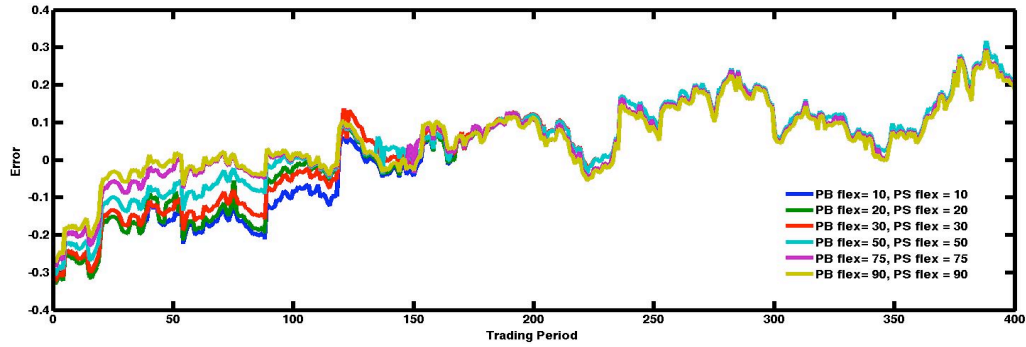
Results B 1: Non-Conservative Buyer vs. Non-Conservative Seller (Set 1). According to this experimental result, see Figure 6.8(a), the flexibility status increases the speed of minimising error. As illustrated in Figure 6.8(b), when both trader have the same flexibility rate, a trader with a high flexibility rate (90) manages to achieve 0 error within the first 50 trading days (yellow curve). However, a similar trader in a similar setting without the flexibility status component needs a minimum of 270 trading days to reach the same level (Figure 6.7(a), red curve). This shows how a basic feature can help traders to show more reasonable trading behaviour. This result further demonstrates that traders can reduce their errors to 0 even faster when they have different flexibility rates. This result demonstrates that the limitation and incapability of traders to update their strategy continuously will let one trader take advantage of the situation and to continuously being a winner of the deals.

Results B 2: Non-Conservative Buyer vs. Non-Conservative Seller (Set 2). Figure 6.8(c) and 6.8(d) show that a trader with flexibility of 10 and a trader with flexibility of 90 (opposite flexibility) can achieve a better price (closer to market price) reaching an error rate of 0 after less than 10 trading days (light blue curve). Reviewing the agreed prices in Experiment B, set 1 (Figure 6.8(a)) and set 2 (Figure 6.8(c)) show that the agreed prices converge to almost one price at one point of the simulation (data points number 170 onward). This is in contrast with the result from Experiment A where prices do not converge (see Figure 6.6). Although the price conversion is in contrast with Experiment A's findings, the agreed prices before the conversion replicate the Experiment A result by demonstrating a negative correlation of flexibility rate and CDS price and a positive correlation of flexibility rate and error rate (Figure 6.8, flexibility rate 10 and 90).

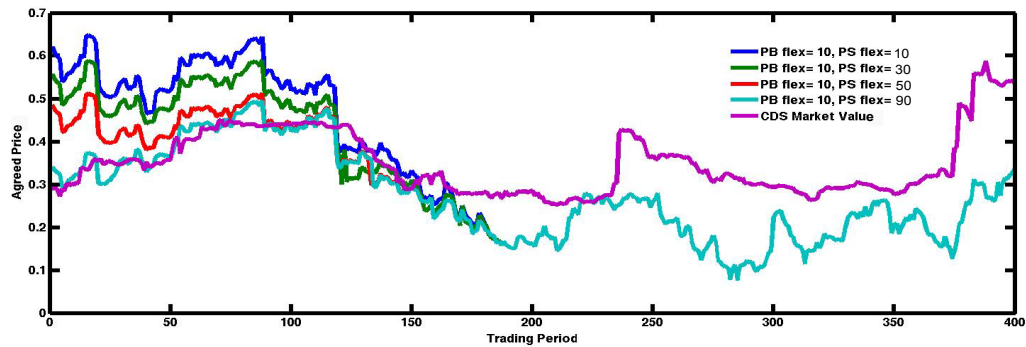
The current model only allows traders to decrease their flexibility rate. This is due to the nature of the CDS pricing and negotiation process. As explained in Section 6.4.2, the protection seller always tries to over-price while the protection buyer tries to under-price the CDS contract in order to profit. Therefore, entering the negotiation process means they should both decrease their flexibility rate so that the protection seller decreases his price and protection buyer increases his the price. Figure 6.9(a) and 6.9(c) illustrate the fluctuation of the flexibility rate for two sets: 1 and 2. In set 1, the protection buyer has a low flexibility of 10 while the protection seller has a high flexibility of 90. In set 2, both protection buyer and seller have a flexibility rate of 90. The main result of two different settings is the speed of minimising the error. However, after reaching a 0 error, Figure 6.8(b) (datapoint 6) and Figure 6.8(b) (datapoint 40), the curves follow a similar pattern in both settings. Another interesting point is that traders lose their capability of minimising the error after datapoint 190 (Set 1) and 200 (Set 2). This can be explained through Figure 6.9(a) and Figure 6.9(c). The traders flexibility rate stays steady after datapoint 190 (Set 1) and 200 (Set 2). This steadiness leads to a bigger error rate since traders do not have the opportunity to provide a better price due to the fixed flexibility rate. The reason for getting trapped in a fixed flexibility is that the current flexibility rate is good enough to successfully make a deal without any negotiation, therefore the trader's flexibility rate stays unchanged for the next time.



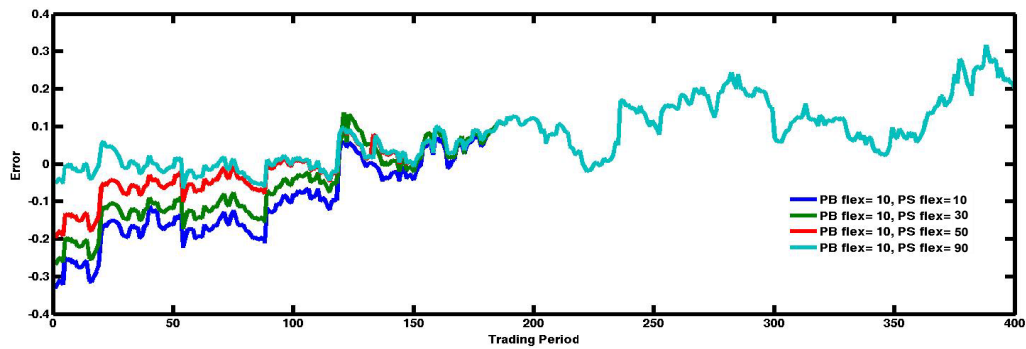
(a) Exp. B - Set 1 (Agreed price): Protection buyer and protection seller have same flexibility rate of 10, 20, 30, 50, 75, and 90.



(b) Exp. B - Set 1 (Pb Error): Protection buyer and protection seller have same flexibility rate of 10, 20, 30, 50, 75, and 90.

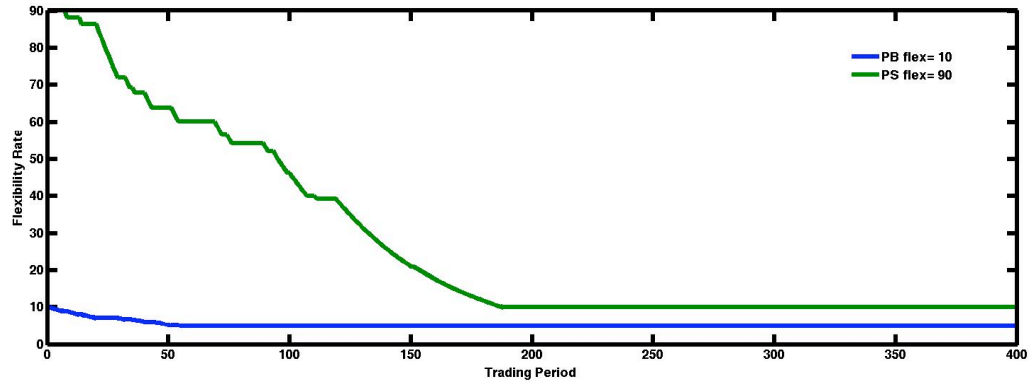


(c) Exp. B - Set 2 (Agreed price): Protection buyer has a fixed flexibility rate of 10 while protection seller has flexibility rate of 10, 30, 50, and 90.

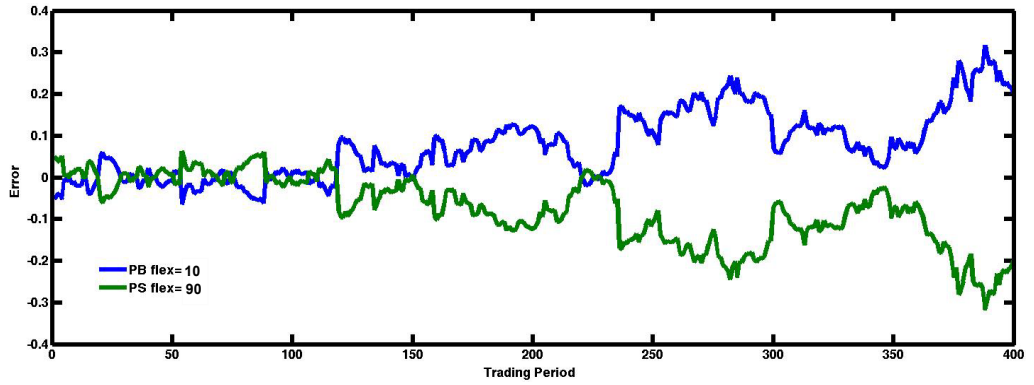


(d) Exp. B - Set 2 (Pb Error): Protection buyer has a fixed flexibility rate of 10 while protection seller has flexibility rate of 10, 30, 50, and 90.

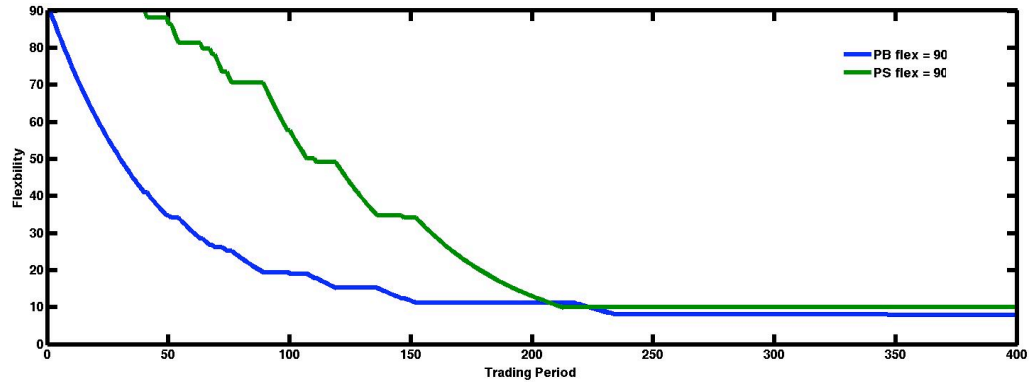
Figure 6.8: Non-Conservative Buyer Vs. Non-conservative Seller: traders agreed price and protection buyer error rate according to different flexibility rate.



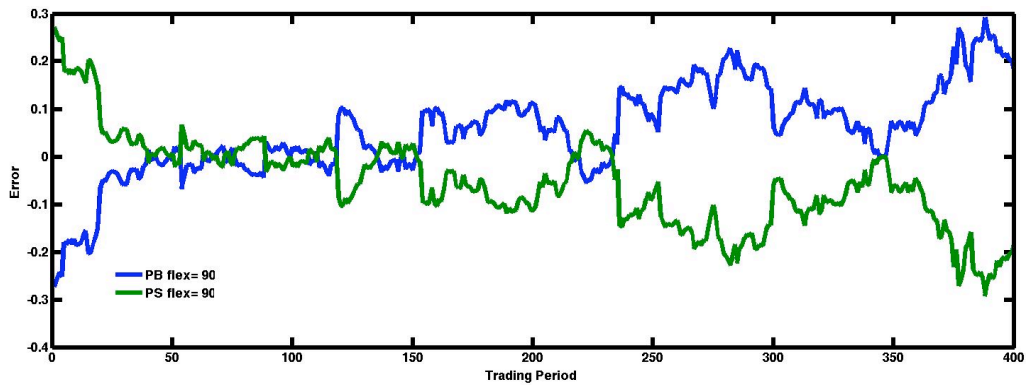
(a) Exp. B - Set 1 (Flexibility rate): Protection buyer flexibility rate of 10 and protection seller flexibility rate of 90.



(b) Exp. B - Set 1 (Error rate): Protection buyer flexibility rate of 10 and protection seller flexibility rate of 90.



(c) Exp. B - Set 2 (Flexibility rate): Protection buyer flexibility rate of 90 and protection seller flexibility rate of 90.



(d) Exp. B - Set 2 (Error rate): Protection buyer flexibility rate of 90 and protection seller flexibility rate of 90.

Figure 6.9: Exp B: Flexibility and Error Fluctuation

6.6 Experiment 2: Naive and Smart Category

6.6.1 Experimental Datasets, Settings

Experiment 2 contains two sets of experiments, A and B, to monitor traders business activities in different strategies and deals. Both the naive and smart strategies are used that were explained in detail in Section 6.4.2 . The real market data is used to simulate the risky companies. The traders flexibility status is set to on to enable a better negotiation process. The traders capital is set randomly at the beginning of each run. Traders choose their strategy randomly at the begging of the simulation. The experiments are run thirty times over 2000 datapoints (trading day). Table 7.1 summarises the agent-based CDS market settings.

Table 6.4: CDS Market Experimental Model Setting

No. of datasets:	20	Type of datasets:	real data (2000 data points)
No. of traders:	16	Type of traders policy:	naive, smart
No. of risky co.:	100	Type of market events:	structural, reduced form
No. of runs:	30	Trader capital range:	10,000,000 - 900,000,000

Experiment A: Impact of Different Policies on Business. This experiment focuses on the impact of different policies on the traders business. It includes three sets of experiments. In experiment set a, all traders use a naive policy. In experiment set b, all traders use a smart policy. In experiment set c, traders will use a smart policy with a probability of P_x ($P_x = 1/dp$, where dp is the number of available policies.), otherwise they select the naive policy.

Experiment B: Impact of Different Deals on the Business. This experiment is focused on the second objective so the traders business is monitored under the mixed policy (chance of selecting each strategy is 50%). The main point of this experiment is to investigate what kind of deals might make problems for traders and what can help a trader to survive longer.

6.6.2 Result

As explained in section 6.3, the agent-based CDS model generates risky companies and traders as soon as their number falls below the minimum boundary. The simulation was begun with sixteen traders and the behaviour of just these sixteen was analysed to guarantee that all the traders analysed had the same market experience. The sixteen traders were equally split into the four wealth categories so that there were initially four traders in the not wealthy, average wealthy, wealthy and super wealthy category.

Result A: Impact of Different Policies on Business. Table 6.5 and 6.6 as well as Figure 6.10(a) and 6.10(b) and 6.10 show the distribution of the trader defaults. The first rows in Table 6.5 and 6.6 show how many of the overall defaults belonged to the different categories. For instance, 43 % of the overall defaults were from not wealthy traders in experimental set a. The following three rows split this number further down into the three periods showing the percentage of overall defaults that occurred in a wealth category in a period. For example, 28 % of all the defaults happened in period 1 by not wealthy traders. The two figures 6.10(a) and 6.10(b) show these numbers visually in a bar chart.

The result from experimental set a using the naive strategy illustrates that the probability of surviving in the society depends on the amount of trader capital. As presented in Table 6.5 and Figure 6.10(a) the traders capital has a considerable impact on the traders survival time. For instance, when considering the not wealthy (NW) and average wealthy (AW) categories (dark blue and cyan in Figure 6.10) one can observe that they are responsible for the majority of defaults of which most happen in period 1 and 2.

In numbers it shows that 28% of the defaults are due to not wealthy traders in the first simulation period while 0% of super wealthy traders defaulted in the same period. Moreover, 43 % of defaults are due to the not wealthy traders while the super wealthy traders are responsible for only 11 % of all defaults.

In contrast to the naive policy, when all traders use the smart policy (set b, Table 6.6 and Figure 6.10(b)) the distribution of defaults is more evenly spread and does not depend as strongly on the initial wealth category. This phenomenon is due to the prioritisation process which gives more priority to buying a CDS compared to selling a CDS contract or lending money to a risky company. Using the smart policy, only 13% of defaults are due to the not wealthy traders in the first simulation period and in period 2 traders from the super wealthy category are responsible for the same amount of defaults as the average wealthy traders.

Table 6.5: Traders Survival Time: Exp. A set a

	NW	AW	W	SW	Total
All Periods	43%	28%	18%	11%	100%
1st Period	28%	14%	7%	0%	49%
2nd Period	15%	12%	5%	4%	36%
3rd Period	0%	2%	6%	7%	15%

Table 6.6: Traders Survival Time: Exp. A set b

	NW	AW	W	SW	Total
All Periods	29%	22%	32%	17%	100%
1st Period	13%	10%	8%	1%	32%
2nd Period	14%	9%	14%	9%	46%
3rd Period	2%	3%	10%	7%	22%

With regard to the impact of the trader policy on the risky companies, the result demonstrates that risky companies are more likely to be rejected in the forced smart policy in comparison to the forced naive policy. This is logical since the smart policy gives a low priority to the loan deals.

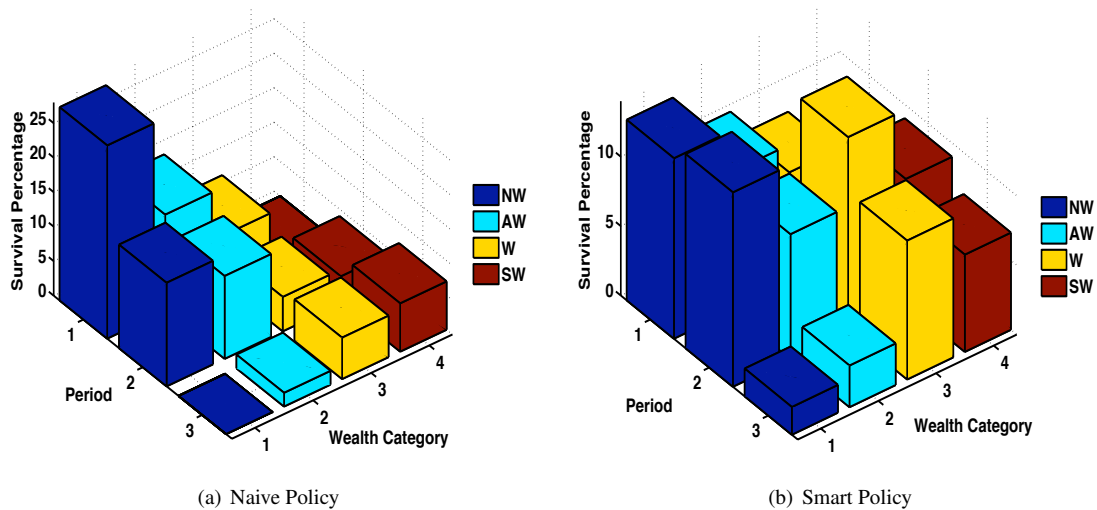


Figure 6.10: Trader Survival Times.

Table 6.7 shows the details of one randomly picked run of experimental setting c. In this setting traders were allowed to choose their strategy at the beginning of the simulation. In this run around 55 % of traders with a *naive* policy (policy = 0) default while around 50 % of the traders with a *smart* policy default. Furthermore, in this concrete instance traders with the *naive* policy started in the category AA while traders with the smart policy started in category A, meaning that naive policy traders had a higher initial capital providing them with an advantage. So in this instance, the *smart* policy allowed the traders to survive better than the *naive* policy traders with less initial capital.

Table 6.7: Summary of Trader's Business Activities: Experiment A set c (the *H* stand for healthy and *S* stand for the structural form default. Policy 0 refers to naive policy and policy 1 refers to smart policy.)

TN	CreditRate	Loan	BuyCds	SellCds	Policy	Status	Def. Date
1	AA	16	14	22	0	S	1044
2	A	16	16	10	1	H	2000
3	A	19	18	24	1	S	1772
4	A	20	20	11	1	H	2000
5	A	11	11	27	1	H	2000
6	AA	18	17	13	0	S	1232
7	AA	17	15	23	0	S	1501
8	A	14	13	20	1	S	1436
9	AA	21	19	14	0	S	1228
10	AA	26	26	20	0	H	2000
11	A	21	20	16	1	S	1864
12	AA	36	36	22	0	H	2000
13	AA	25	25	24	0	S	1498
14	AA	19	19	23	0	H	2000
15	AA	9	9	6	0	H	2000

Results B: Impact of Different Deals on the Business. Visualising the traders money transactions, see Figure 6.11, shows that the main decrease in the traders capital amount is due to lending money. Additionally, observing the traders default time suggests that traders who did not default in the first 1200 days are those which firstly lend less money (smart traders) and secondly did buy protection for all the loan deals that they have to reduce the risk of risky companies default. See Table 6.7 and traders number: 2, 4, 5, 10, 12, 14, and 15 which protected each loan by buying a CDS contract. Another interesting observation is that those traders who default most are those that sold more CDS protections. Finally, one can observe that six out of eight traders who defaulted, defaulted in the turbulent time of the market. See Table 6.7 and Figure 6.12 for traders number: 11, 3, 7, 6, 8, 1, 9.

Although the weak traders are most exposed to market events especially in a crisis period the trader number 2 successfully managed to survive. Trader number 2 stayed alive by choosing a smart policy, accepting a relatively small number of loan deals, protecting all its loan deals by buying CDS protection for them and selling a relatively small number of CDS contracts. See Figure 6.11 and Table 6.7 .

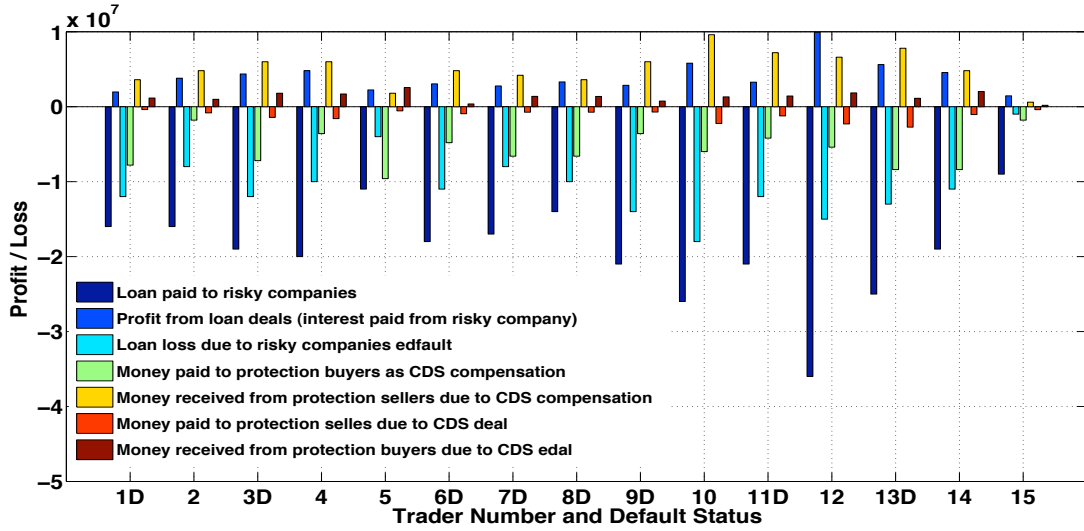


Figure 6.11: Overall Trader's Profit and Loss

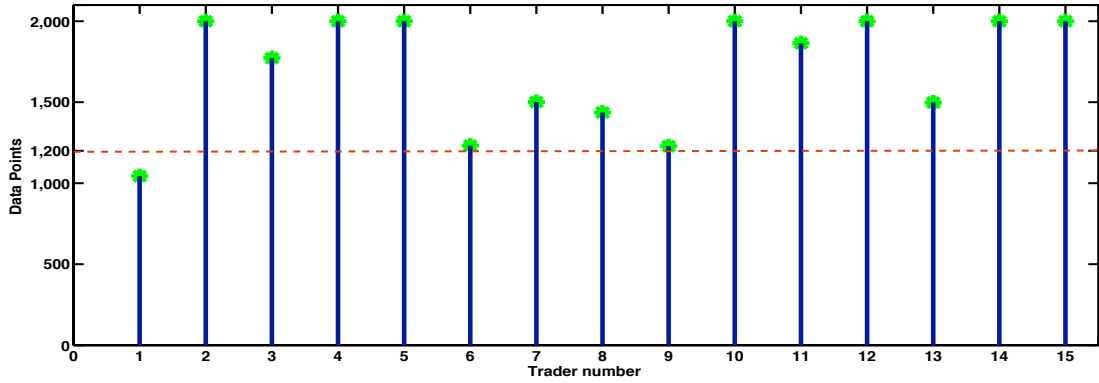


Figure 6.12: Trader's Survival Time

6.7 Experiment 3: Risky and Reduced-risk Category

6.7.1 Experimental Datasets, Settings

Experiment 3 contained two sets of experiments, A and B, to monitor traders business activities under different strategies and deals. Likewise Experiment 2, the real market data was used to simulate the risky companies and the traders flexibility status is set to on. The traders capital is set randomly at the beginning of each run and traders choose their strategy randomly at the beginning of the simulation. In these experiments, traders use the advanced strategies which were explained in detail in Section 6.4.2. The experiments are repeated thirty times over 2000 datapoints (trading day). Table 6.8 summarises the agent-based CDS market settings.

Experiment A: Impact of Different Strategies on the Traders Business. Experiment A consists of three sets where in experiment set a, all traders use a risky strategy. In experiment set b, all traders use a reduced-risk strategy. In experiment set c, traders may use a reduced-risk strategy with a probability of P_x ($P_x = 1/dp$, where dp is the number of available strategies.), otherwise they use a risky strategy.

Table 6.8: CDS Market Experimental Model Setting

Nr. of datasets:	20	Type of datasets:	real data (2000 data points)
Nr. of traders:	50	Type of traders policy:	risky, reduced-risk
Nr. of risky co.:	1500	Type of market events:	structural, reduced form
Nr. of runs:	30	Trader capital range:	5,000,000 - 80,000,000

This is to make a balanced market with mixed strategy.

Experiment B: Impact of Different Deals on the Business. In Experiment B the focus was on the second objective of monitoring the traders business under the different settings: set a, set b, and set c (which are performed in Experiment A). The main point of this experiment is to investigate what kind of deals might make problems for traders and what can help a trader to survive longer.

6.7.2 Result

Table 6.9 presents the model statistics over 30 runs using real world data. The pie chart in Figure 6.13 summarises the third row which shows the number of sold CDS contracts depending on five experimental settings. The three pie charts in Figure 7.14 correspond to the last three rows and show the dependence of trader defaults on the experimental setting.

Table 6.9: Summary of Experiment 1 Result

Result Per Run	set a (No CP Risk)	set b (CP Risk)	set c (Combined)	set c (No CP Traders)	set c (CP Traders)
Nr. of Loan Deals	3284.70000	3260.60000	3311.6000	1592.4	1719.2
Nr. of CDS Deals (Buy)	1270.90000	788.90000	976.60	534.8	441.80
Nr. of CDS Deals (Sell)	1270.90000	788.90000	976.60	590.6	386
Nr. of Defaults	4.7 (9.4 %)	2.6333 (5.27 %)	3.2 (6.4 %)	1.6 (out of 24.2; 6.61 %)	1.6 (out of 25.8; 6.2 %)
Credit Rates AA Category per Trader	0 %	0.42 %	0%	0 %	0 %
Credit Rates A Category per Trader	4.8 %	0.41%	0.8 %	0 %	1.562 %
Credit Rates B Category per Trader	36.9 %	24.667 %	32.6%	33.33%	31.81 %

Result A: Impact of Different Strategies on the Traders Business. Table 6.9 illustrates that when all traders use a risky strategy (set a), then the overall rate of trader defaults increases. As it is mentioned in Section 6.4.2, the difference between the risky strategy and reduced-risk strategy is the sensitivity of the traders to counterparty risk. The results show that the number of traders who care about the counterparty risk is inversely proportional to the number of traders who default and the increase in the number of traders who use a reduced-risk strategy leads to a market with fewer traders defaulting. The pie chart demonstrates this same point visually. The No CP Risk setting represents by far the biggest segment in the pie chart.

The picture becomes more pronounced when categorised according to the initial credit rating as done in the three pie charts in Figure . The left most pie chart can be neglected since only 0.42 % of traders defaulted overall. The pie chart in the middle corresponds to traders that started in wealth category A and here the lion's share of defaults lies clearly with the No CP Risk category. For traders starting with wealth category B the picture becomes more uniform and the setting has no major impact. A possible interpretation is that some amount of capital is needed to profit from the smart policy.

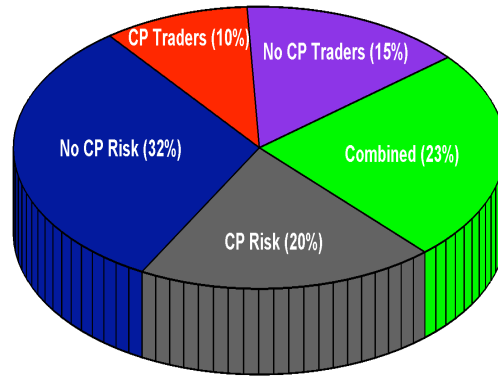


Figure 6.13: Trader Default Based on CDS Sell

An interesting observation is that if the traders that default are categorised based on their initial credit rating category, we can conclude that the traders with a lower credit rate are more likely to default as everyone would like to make deals with good credit rated trades. These results show that in the smart market where the traders are all concerned about their counter party risk, the rate of default in a lower credit rating category increases and this presents the difficulty that the traders with a lower credit rate face for making deals. Experiment set c demonstrates a market with mixed strategies. Our results confirm the result of Experiment set a and Experiment set b in the sense that traders with a lower credit rate show more defaults.

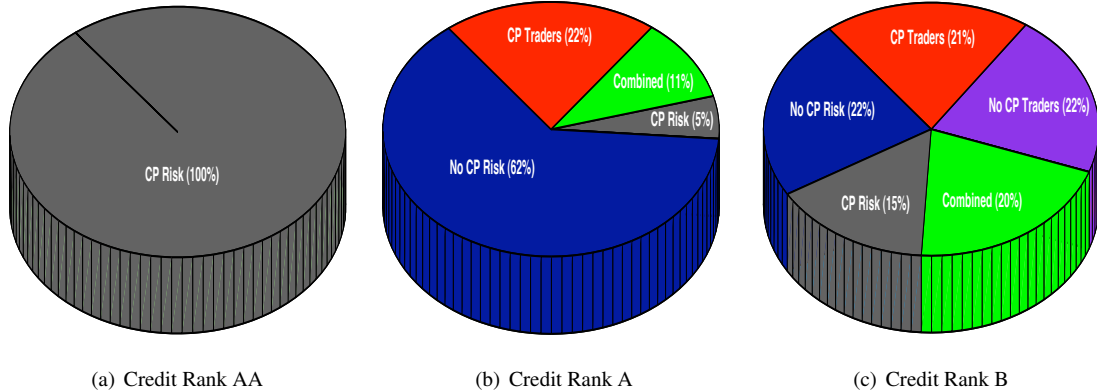


Figure 6.14: Default Based on Credit Rank

Result B: Impact of Different Deals on the Business. Studying the number of deals which traders made under the different market settings (set a, set b and set c) reveals that the increase in the rate of CDS deals (the number of buys/sells) leads to an increase in traders defaulting in the economy. This fact is particularly interesting because the CDS contract is introduced to the market as a protection tool. But as illustrated in our result, the CDS contract causes problems for traders. Traders do not default by lending money to risky companies; they only lend money if they have enough money to lend otherwise they do not lend money. In Experiment set c, it is noticeable that the traders with the risky strategy made more CDS deals compared to the traders with reduced-risk and, as a result, the default of traders with a risky strategy is higher than the default rate of traders with a reduced-risk strategy.

6.8 Evaluation and Discussion

This chapter presented our AB model of CDS market. The trader's behaviour was monitored under different settings. Although this model is not the exact replication of the real CDS market, our model successfully replicated some of the real human behaviour. Our modelled traders demonstrated the power of money and knowledge in the society. As presented in the result, the traders who survive are those who make a better use of information or those, which are wealthy. For instance, the non-conservative traders use the feedback information to better adjust their flexibility rate or traders who use a reduced-risk strategy successfully reduce their default rate by taking the counterpart risk into the account.

Moreover, the wealthy traders have a lower default rate and they usually do not default in the early times of the simulation. This is due to the level of security that money brings to a trader. For example, if two persons both have well-paid jobs but one of them owns a house outright while the other one has a mortgage on a house. If they both lose their jobs for any reason (we witnessed a huge number of redundancies during 2007-2010 financial crisis), one becomes only unemployed while the other one becomes unemployed and also exposed to the risk of losing his house.

To better validate and illustrate the capability of this CDS Market Experimental Model in dealing with the application to the real world, the next chapter provides our final version of AB CDS market where we thoroughly investigate the CDS market and compare our findings with the real world.

6.9 Conclusions

Traditional macroeconomic forecasting models have demonstrated incapability in forecasting market events, suggesting that there is a serious need for more precise and practical modelling of complex economic systems. In particular, a model which focuses on the credit default swap market - a market which trades a type of financial product which helped precipitate the global crisis - would provide an important tool to help policy makers avoid such a crisis in the future.

In this chapter, we introduced a CDS Market Experimental Model using a multi agent approach. Our CDS Market Experimental Model simulates the credit default swap market and aims to provide an efficient experimental tool for investigating the individual traders and the emergent market behaviour as well as studying the impact of different policies and controls on the whole system. To our knowledge, this work is the first attempt toward simulating the CDS market using agent based modelling. Our model successfully replicated some of the characteristics of the real world financial market.

We studied the market under different scenarios using real world data. For instance, we showed that the trader's policy and flexibility has a direct impact on their offered price and also final agreed price. Moreover the traders showed a competitive behaviour when their policy allows them to compete. Our result suggested that the smart or wealthy traders are the survivors of the economy, thus having more opportunity to improve their financial situation replicating the fact that "the rich get richer and the poor get poorer". Moreover, we found that those traders who used "risky" strategies and did not consider counterpart risk were more likely to default compared to the traders who used "reduced-risk" strategies. We also showed that the traders who sold more CDS protection were those who defaulted more. Based

on the findings the agent-based CDS model successfully helped us to monitor the impact of different policies in the market and to explain the result of events such as traders default and rejection of risky companies loan requests. The AB model managed to replicate some of the real world behaviour such as difficulties in getting a loan in a smart society, survival difficulties that poor traders face and the problem of selling a CDS contract without having enough credit to support the deal. Our results and findings illustrate the capability, efficiency and utility of agent based modelling and its potential as a helpful tool for policy makers.

The next chapter presents a our Cds Agent-Based Virtual Economy and its potential application where we utilise complementary components of a data generative model and CDS pricing. We will illustrate how our three novel approaches can cooperate to provide a feasible tool for studying CDS market.

Chapter 7

CDS Agent-Based Virtual Economy

The 2007-2010 financial crisis highlighted the danger of relying on a single analytical tool to observe financial markets and a serious need to develop complementary tools to improve the robustness of the overall financial framework. Scientists in the different fields of physics, engineering, psychology, and biology developed sophisticated tools for analysing complex dynamic systems in a rigorous way. These models have proven helpful in understanding many important but complex phenomena such as epidemics, weather patterns crowd psychology and magnetic fields. However, in the financial context, these tools have been rarely used [Trichet, 2010].

Chapters 4, 5, and 6 concentrated on the three primary challenges of lack of sufficient data to support research, lack of efficient Credit Default Swap (CDS) pricing technique to be integrated into agent based model, and lack of practical CDS market experimental model, that are faced by designers of a CDS market. It was explained how these challenges were tackled by taking advantage of well established techniques in the fields of statistics, evolutionary biology and Agent Based (AB) modelling. The three identified challenges are investigated individually and the approaches used here can be seen as modular building blocks that can be applied to a variety of applications. However, the true value of these approaches can be enhanced if they work together.

For instance, by catering our AB CDS market with simulated data the restriction of data availability can be overcome thus providing open end simulations. Data for different scenarios can also be generated thus making it possible to monitor market behaviour under different conditions. In addition, traders can improve their pricing technique if they use the derived CDS pricing solutions provided by the CDS approach instead of the theoretical benchmark. This approach can improve the traders overall performance thus increasing the AB models utility.

The motivation of this research is to identify and tackle the steps needed to create a general purpose model for investigating the CDS market. This chapter presents how the cooperation of three proposed solutions in Chapters 4, 5, and 6 can prove to be valuable in an Agent-Based Computational Economics (ACE) application. It will be demonstrated how the proposed solutions can collaborate to overcome the limitation of existing models and to form an AB model which is more close to reality, thus more practical in dealing with real world challenges.

This chapter consists of eight sections. The first section illustrates the overall model (Section 7.1).

This is followed by presenting the experimental objectives (Section 7.2) and datasets (Section 7.3). Three experiments are introduced and explained in Sections 7.4, Section 7.5, and Section 7.6. Section 7.7 evaluates and discusses our results. This is followed by presenting our conclusions in Section 7.8.

7.1 Overall Model

Our overall model consists of our three approaches explained in Chapters 4, 5, and 6. Figure 7.1 is a conceptual illustration of the overall model called Credit Default Swap Agent-Based Virtual Economy (CAVE). The aim is to integrate or link the three approaches together to form a comprehensive experimental tool.

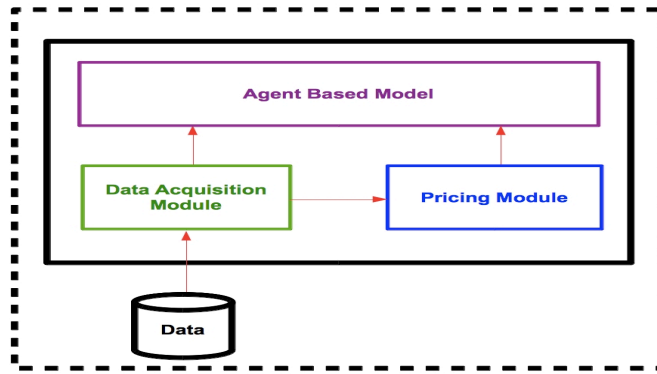


Figure 7.1: Overall Model

This section provides an overview of the implementation details of each approach.

Data Generative model. The data generative model is the statistical approach which is implemented in MATLAB. MATLAB is a numerical computing environment which allows matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages, including C, C++, Java, and Fortran.

The data generative module has three inputs and one output. The model inputs are the available historical bond prices, a market trend, and the length of the simulated data. The data generative module takes the historical data (e.g. company bond prices and interest rate) and the requested lengths of the simulated data. The output of this module is simulated data of the requested length which is based on characteristics of the historical data.

As explained in Chapter 4, the data should be categorised into different parts if it shows different behaviour in different periods of time. This helps inferring the data characteristics as closely as possible. In these experiments, the data is divided into two categories *Normal time* and *Crisis time*. This allows us to introduce the crisis period at any data point that is required and as many times as required. This is extremely beneficial since different scenarios can be created. The data acquisition module stores the simulated data in a text file thus any other program can access this information directly.

CDS Price Calculator. The CDS price calculator, called pricing module hereafter, is the evolutionary

approach which is implemented in the C language. The pricing module takes the historical data (e.g. bond prices, CDS prices, and interest rate) as input and stores the derived solutions, the chromosomes, in a file. The stored chromosomes can be accessed by any program. However, the chromosomes should be translated and the output should be processed based on the original Cartesian Genetic Programming (CGP) parameters.

To make this possible, a chromosome reader function was derived from the original CGP program. The function takes the chromosome, a parameter file, and data (e.g. bond prices and interest rate) as inputs. It calculates the CDS price based on these and returns the CDS price as the output. This function has been added to the CAVE model and traders can call this function in order to calculate the price of the CDS.

Agent Based Model. Our AB model has been modified in order to make the following choices possible:

1. The data feed can be set to 0 or 1 by the experimenter at the beginning of the simulation. Data feed 0 refers to real data and 1 refers to simulated data.
2. The pricing strategy can be set to 0, 1, or 2 by the experimenter at the beginning of the simulation. Where 0 refers to the Duffie pricing approach, 1 refers to the CGP approach, and 2 allows the trader to choose between Duffie and CGP.

The experimenter can set these parameters to decide which data source and what kind of CDS price calculators should be used in the simulation.

7.2 Experimental Objectives

The primary objective of this chapter is to demonstrate how the cooperation of our three proposed solutions from three different fields of science can improve the practicability of existing model. In this chapter the model is evaluated by focussing on four categories of experiments. Each category focuses on some specific setting of our model. The following objectives are being investigated through our experiments:

1. **Impact of simulation time and simulated data on overall investigation.** The main interest is to see whether the generative model, and therefore the open-ended simulation as well, has the capability to generate data for different scenarios can improve our investigation.
2. **Impact of complementary tools on trader behaviour.** It is interesting to see whether the complementary tools such as the CDS price calculator can improve trader performance or not.
3. **Capability of CAVE in Identifying Risky Deals.** It is of interest to observe whether Agent-Based Modeling (ABM) models such as CAVE can help to explain the reason behind the market events such as the default of traders.
4. **Capability of CAVE in studying different policies.** The main question is how experimental tools such as CAVE can help to avoid mistakes?

Statistical tests are used in this chapter to verify that certain effects that are observed are significant in the statistical sense and not due to chance. In particular, a so called two-sample test is used to verify with high probability that two sample sets are generated from different distributions (See [Lehmann and Romano, 2008] for full details of this technique). These two distributions will typically occur in the following set up: there is a baseline setting from which to collect data and then a second setting in which a crucial variable of the system is modified. The data is examined to find if this modification changes the behaviour of the system and the test will give a probabilistic guarantee for this.

Statistical tests work by disproving hypotheses. The hypothesis is here that the two distributions are the same and if the test can reject this hypothesis with high probability then there is a guarantee that a real effect is being observed. The test being used is the classical Student's t-test which is the widely established method to perform such comparisons. The t-test has a parameter α which is the significance level. In this chapter the test is against a significance level of $\alpha = 5\%$, that means if the test rejects the hypothesis of equal distributions for this α then it is $95 + \%$ sure that a real effect is being observed. The t-test also gives the exact value to which it can reject, that might be something like $2.321 \dots \%$, though, it is statistical practice to test against a fixed α and to just report this exact value for further interpretation and confidence in the result. Every statistical test comes also with assumptions. The most important class of statistical tests is sometimes called large-sample tests since it assumes that enough data has been recorded so that the distribution of the sum of the values approximates the normal distribution well. This effect will happen under rather weak assumptions due to the central limit theorem, though, there needs to be enough data and these tests are, therefore, called large sample tests. 50 samples are recorded per experiment which is rather large. The Student's t-test is such a large sample test which also has the assumption of equal variance of the two distributions.

The next sections explain the details of the three experimental categories and the associated results. We start by discussing the impact of our data generative module on our overall market understanding. Then we present how our CGP pricing approach improves the overall performance of traders by reducing the default rate. Finally we demonstrate our CAVE model capabilities in identifying the market problems and studying different policies and regulations.

7.3 Experimental Dataset

Three different sets of data are prepared for the experiments of this chapter. Each set includes two types of data: company bond prices and interest rate. The first set, called *Real Market*, consists of twenty company bond prices as well as one set of interest rates. The real market data has 2000 data points (trading days) and features market characteristics in both normal time and turbulent time (see Chapter 4, Section 4.1 and 4.2 for full description of our real data.).

The second and third sets are simulated using our data generative model explained in Chapter 4, Section 4.3. The independent model is used (see Section 4.3.3) to simulate interest rates and the dependent model (see Section 4.3.4) to simulate risky company bond prices. The simulated data consists of 5000 data points.

The second set, called *Normal Market*, is simulated using the normal time data characteristics. The

third set, called *Crisis Market*, is simulated based on both normal time and crisis time data characteristics. The crisis market includes two crisis periods: data points 1600 to 2000 and data points 3000 to 3500.

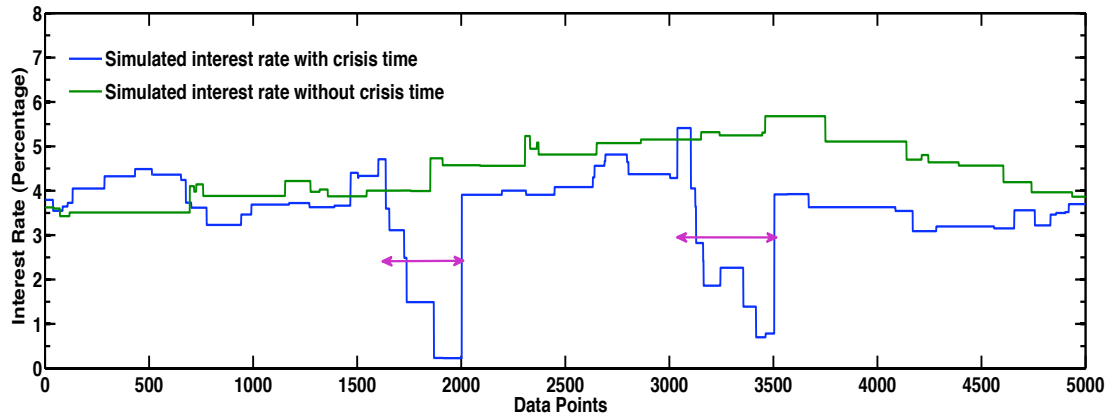


Figure 7.2: Simulated Interest Rates Using Real Interest Rate Data Characteristics (The purple double arrows represent the crisis periods).

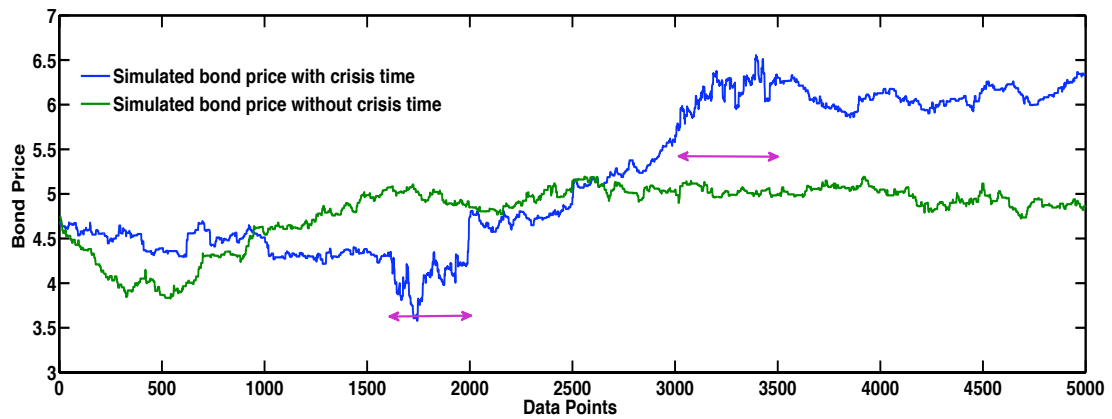


Figure 7.3: Simulated Bond Prices Using One Real Company Data Characteristic (The purple double arrows represent the crisis periods).

One sequence of interest rates is sampled and 600 company bond prices (30 samples from each real world company) for the normal market and the crisis market. Figure 7.2 and Figure 7.3 illustrate our sampled interest rate and bond prices for both normal market (simulated data without crisis time) and crisis market (simulated data with crisis time).

7.4 Experiment 1: Impact of Time and Data

7.4.1 Experimental Settings

One of the important issues when studying events such as a company default or a financial crisis is that these events are rare. Therefore, it is not usual to have enough historical information to help the investigations. In addition, having limited historical data does not allow the study of future possibilities and different scenarios.

As discussed in Chapter 4, the availability and limitation of real data restricts the results and therefore the reliability of the resulting insights. A realistic data simulator helps in simulating rarer events using the limited information on hand as well as enabling the investigation of different scenarios. In this experiment, the trader performance (default rate) is studied under three different scenarios (*real market*, *normal market* and *crisis market*). The trader defaults are based on the structural approach to better study the traders' performance. Considering all the traders have access to the same information and pricing technique, the initial capital of the traders is the only feature which varies from trader to trader thus traders have different credit rates (see Section 6.4.1 for more details on our credit rating system) at the beginning of the simulation.

The trader default rates are monitored and their interest for selling CDS contracts under different scenarios. In these experiments reveal how the data characteristics and simulation period can help in gaining a better understanding of the market and individual trader's behaviour. Table 7.1 shows the general experimental settings. The traders capital is set randomly at the beginning of each run. All experiments are repeated 50 times.

Table 7.1: CAVE General Settings

Nr. of datasets:	20, 600	Type of datasets:	real data(2000 data points), simulated data (4000 data points)
Nr. of traders:	30	Type of traders policy:	reduced-risk
Nr. of risky co.:	200	Type of market events:	structural, reduced form
Nr. of runs:	50	Trader capital range:	5,000,000 - 80,000,000

7.4.2 Results

Table 7.2 shows trader defaults averaged over 50 runs for each of the three datasets. On average there is a decrease of 0.8 trader defaults per run in the normal market in comparison to the real market based on the results. Though, as explained in Section 7.3, the simulation period in the normal market setting is double the length of the real market data. Hence, in the real world data a significantly higher number of defaults per time is observed compared to the normal market setting. This is not surprising since the real market includes a considerable crisis period of 400 data points.

Table 7.2: Trader Default Rates

	Real Market	Normal Market	Crisis Market
Mean	15.68	14.8	20.34
STD	2.6064	2.5071	1.6734

The difference in the average number of defaults between the normal market setting and the crisis market setting emphasises the importance of the crisis times further: now both data sets stretch an equal time period, though, in the crisis market it is observed that there is on average 5.54 defaults more than in the normal market. This is around one third of all defaults observed in the normal market model. The crisis market includes two crisis periods which add up to 900 data points in total.

The difference in defaults between the normal market and the crisis market is significant to a level

of $\alpha = 5\%$ in the T-test. In fact the critical value is of the order of 10^{-23} which means that there is far higher confidence than the α level of 5 percent. Therefore, the crisis periods can be highlighted as a significant cause the of trader defaults. The trader defaults per category are normalised by the number of traders in the category.

In this model, traders have different initial capital, therefore different initial credit ratings. To investigate which traders tend to default the traders are categorised based on their initial credit rating. Figure 7.4 illustrates the trader default rates based on their initial credit rates.

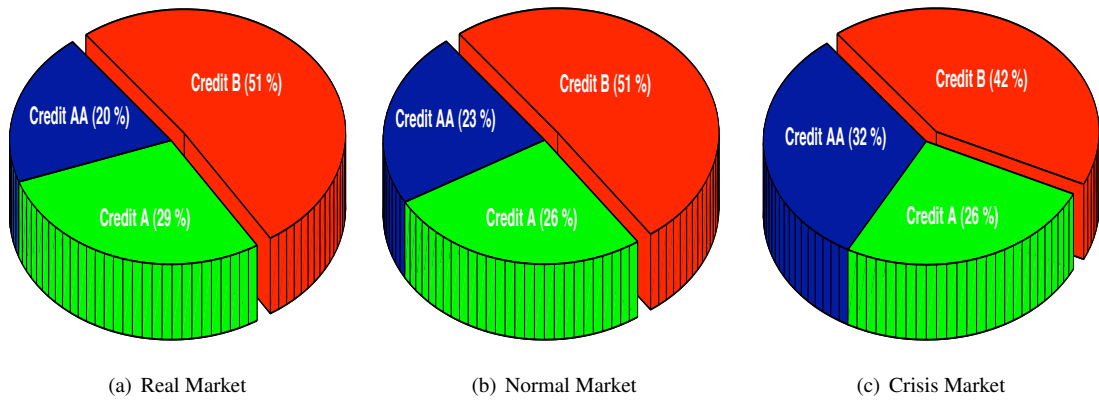


Figure 7.4: Trader Default Rates Based on Initial Credit Rating.

The results from Experiment 1 suggest that there is a direct relationship between credit rank and trader default rates where the wealthy traders tend to survive longer (default less). This experiment confirms that the CAVE model is implying the inevitability of what Karl Marx called the Law of Increasing Poverty [Yi, 1975] when the rich get richer and the poor get poorer over time.

However, considering that traders only default based on the structural approach, this result demonstrates a kind of irregularity where 52% of traders (average per run and over three market data) with B credit rate survive until the end of the simulation (see Figure 7.4(a), 7.4(b), and 7.4(c)). Taking into the account this irregularity, it can be assumed that there is an event, a business activity or strategy which has a high impact on the traders financial situation and can lead to a default event.

In Figure 7.5 and Figure 7.6 the risky company defaults and trader defaults are shown over time to obtain a better insight into the model dynamics. The default events were averaged over 50 runs. The result suggests that there is a direct relationship between company defaults and trader defaults. There is a higher number of trader defaults when there is a higher number of company defaults in the market (see Figure 7.5 and Figure 7.6, data points 1600 to 2000 which is the crisis period for the real data and the crisis time data set).

As explained in Chapter 6 traders have two forms of deals which they can make with risky companies: a loan deal and CDS deals. Making a loan deal and buying a CDS deal cannot lead to a direct default relationship between risky companies and traders because traders give loans only if they have enough money and also they do not pay a deposit when they buy a CDS contract. Thus, it is possible that selling a CDS contract may be the reason for the trader defaults.

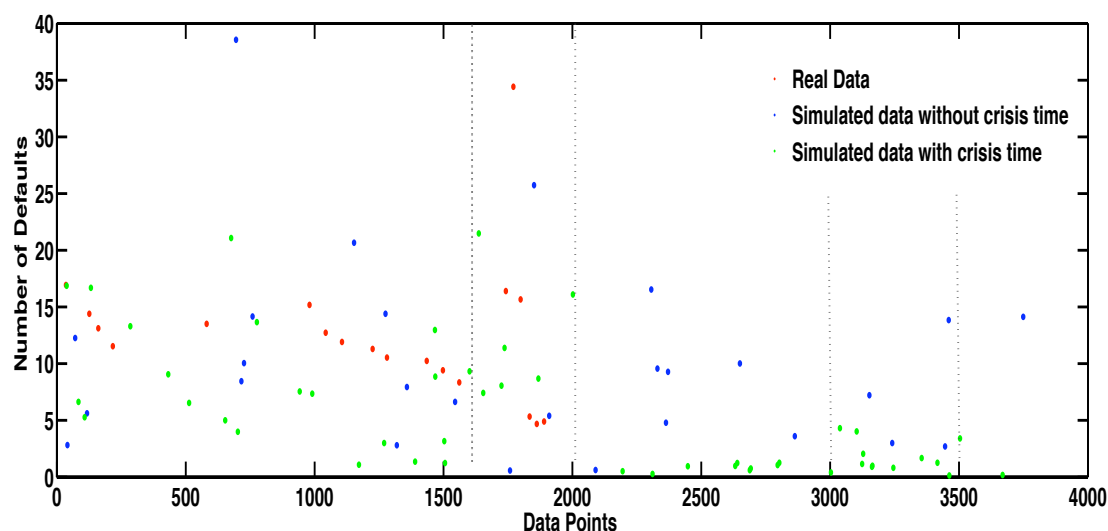


Figure 7.5: Risky Company Defaults

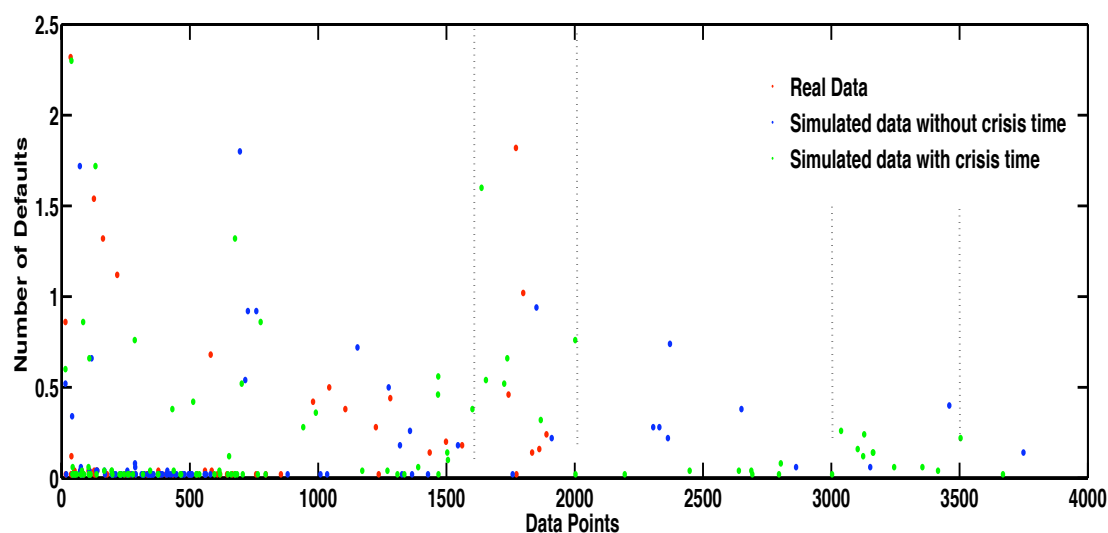


Figure 7.6: Trader Defaults

Table 7.3 presents the overall number of CDS deals which are sold by traders on average over 50 runs. Interestingly, the result shows that traders sold more CDS deals in the normal market compared to the real market and the crisis market. Taking into account that the default rate in the normal market was less than the default rate in the real market and crisis market (see Table 7.2), it can be argued that selling CDS contracts is not in itself dangerous, however further investigation is needed to assess the impact of these contracts on trader business and on the market.

Table 7.3: Overall Traders CDS Sell

	Real Market	Normal Market	Crisis Market
Mean	289.94	529.1	260.56
STD	33.159	20.226	37.503

Figure 7.7 demonstrates the average number of CDS contracts sold by traders considering the initial trader credit rates. The result suggests that traders with a higher credit rating tend to sell more CDS contracts. Since traders use the reduced risk strategy (see Table 7.1) this is the expected behaviour as traders try to buy the CDS contract from traders with good credit ratings. Finally, comparing the results from Figure 7.7 and 7.4, shows that traders with B credit rate, sold less CDS contracts but had a higher default rate. This result highlights the potential risky side of the CDS contract.

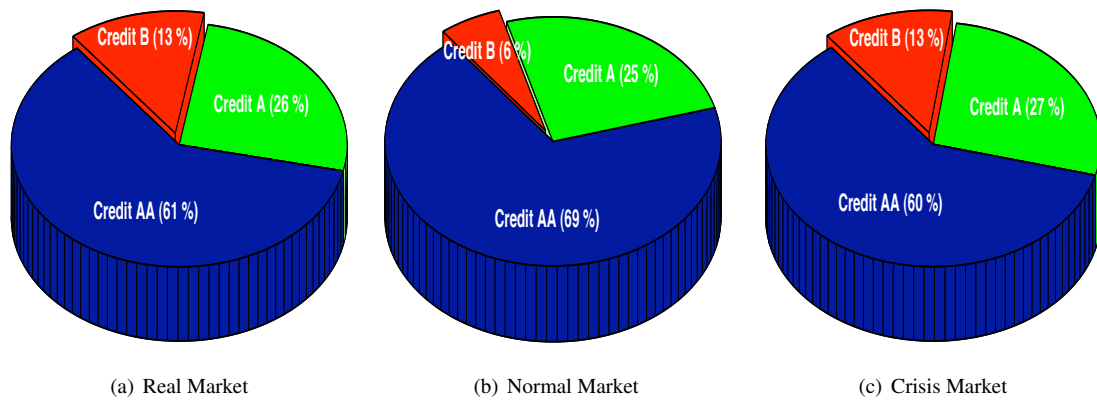


Figure 7.7: Average Number of Sold CDS Contracts Based on the Initial Credit Rating.

The comparison in this experiment demonstrates the advantages of the simulation environment. For instance, it would be easy to analyse the effect of the crisis periods on default rates and on CDS selling rates which would not have been possible based on the real world data alone. Using the data generative model data for different scenarios can be generated thus observing the traders in different situations, increasing the simulation period as needed, and generating samples of arbitrary size. It is also possible to generate many companies to participate in the artificial environment (600 in these experiments), while with the real world data there is a restriction of 20 companies.

7.5 Experimental 2: Impact of complementary tools

7.5.1 Experimental Settings

Chapter 5 introduced the CGP technique as an alternative approach for CDS pricing and its advantages and disadvantages were discussed, also the effectiveness of CGP as a bio-inspired evolutionary method for a complex real-world financial market analysis, and its suitability to be integrated with Agent based modelling. The previous Experiment 1 was repeated with two different pricing techniques: the Duffie approach and the CGP approach to study the impact of the CGP technique.

We use the CGP pricing formula (chromosomes) which is derived from our pricing module explained in Section 5.1. This formula is derived using the real world data consisting of twenty companies (Discussed in Chapter 4). Data was collected from 1st January 2004 till 25th June 2009 (which includes the recent highly turbulent period of the markets). We used 5-fold cross-validation for selecting our training and test dataset among 40,000 original samples. Table 7.4 demonstrates our CGP model experimental setup. We ran all experiments with the same settings. The number of generations and the number

of nodes vary in different experiments. A simple function set is chosen, containing only fundamental operators as listed in Table 7.4. In addition to our financial inputs, three constant integers (1, 2 and 3) have been given as inputs to the model as well. We selected the best chromosome (Fitness 6905.8) for this experiment.

Table 7.4: CGP Settings

General Setting		Function	Symbol
Population size:	5	Add	+
Mutation rate:	0.70	Subtract	-
No. of generations:	150000, 400000	Multiplication	*
No. of runs:	20	Division	/
No. of rows:	1	Power	Pow
No. of cols:	40 or 50	Square root	Sqrt
Levels back:	40 or 50		

The same setting as in Experiment 1 was applied. For the purpose of this experiment, once the Duffie pricing method and once the CGP pricing method were used by traders. The trader behaviour is being monitored observing the number of deals that they make and the trader default rates.

7.5.2 Results

The default rates are shown in Table 7.5. In comparison to the default rates in Experiment 1 a very significant decrease in defaults on average can be observed. That is for the real market (down from 15.68 to 5.58), for the normal market (down from 14.8 to 4.34) and the crisis market (down from 20.34 to 4.62). This drop is most apparent for the crisis market and in fact there is now only a minor difference between the default rate in the normal market and the crisis market (not significant for a level of $\alpha = 5\%$).

As expected the difference in defaults between experiments 1 and 2 is highly significant for all three categories. In all three cases a significance level of $\alpha = 5\%$ is easily reached. In fact, the critical values are far better than this (10^{-41} for the real data, 10^{-44} for normal time and 10^{-68}).

Table 7.5: Overall Traders Default Rate

	Real Market	Normal Market	Crisis Market
Mean	5.58	4.34	4.62
STD	1.739	1.5986	1.701

The number of defaults based on the initial credit rating was measured again. There is now a significant shift of defaults from traders with initial credit ratings A or AA to traders with initial credit rating B. Most of the defaults are now in this last category of traders with initial credit rating B. This is a striking difference to what was observed in Experiment 1 where the traders with credit rating B also had most defaults, though in total only around 50% compared to the nearly 90% that are observed now in the simulated data.

An obvious interpretation is here that traders with higher capital can make better use of the pricing tools while traders with low initial capital cannot leverage the advantages of the pricing tools efficiently.

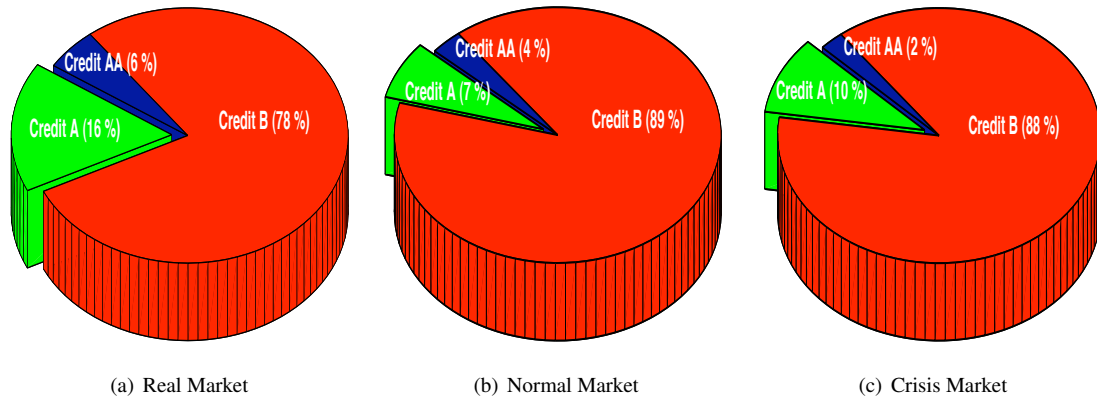


Figure 7.8: Trader Default Rates Based on the Initial Credit Rating.

Like the number of defaults the number of sold CDS contracts also decreased very significantly from Experiment 1 to Experiment 2 (down from 289.94 to 70.21 for the real market, down from 529.1 to 75 for the normal market and down from 260.56 to 64.22 for the crisis market, Table 7.6).

Table 7.6: Number of CDS Sold by Traders

	Real Market	Normal Market	Crisis Market
Mean	70.21	75	64.22
STD	29.459	28.92	11.52

Beside the significant reduction in CDS contracts sold there is also a shift in traders that sell CDS contracts (see Figure 7.9). In Experiment 1 traders with initial credit rating AA sold the lion's share of CDS contracts, while now in Experiment 2 a lot more traders with credit rating A or B are selling contracts. This effect is stable over the three data sets and might hint at another reason for the increase in defaults of traders with initial credit rating B.

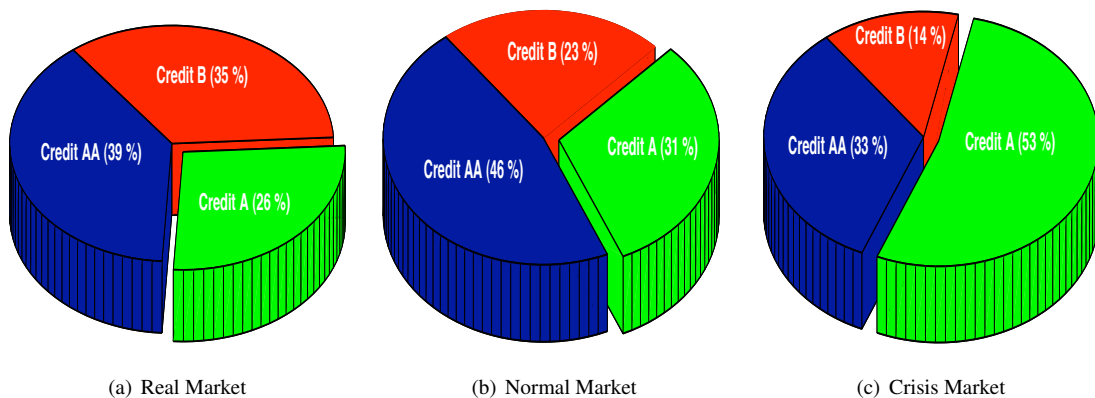


Figure 7.9: Average Number of Sold CDS Contracts Based on the Initial Credit Rating.

The time when traders default was also looked at once again (Figure 7.10). The numbers are now low with two peaks, one close to the starting point of the simulation and one in the first crisis period. Interestingly, the number of defaults goes down significantly in the second period of the simulation

suggesting that the pricing tool has a stabilising effect on the market.

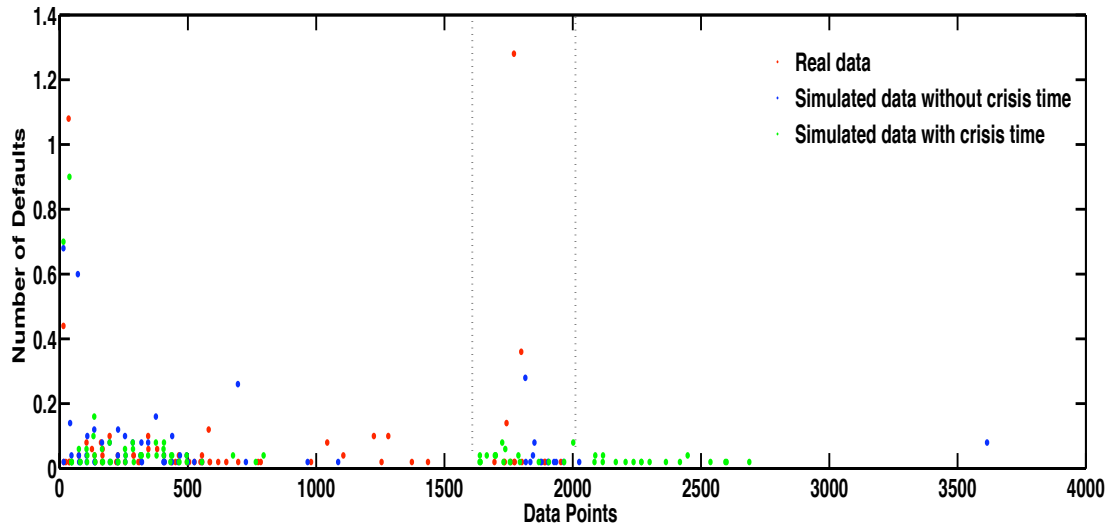


Figure 7.10: Trader Default

Moreover, observing the number of CDS deals that traders make, confirms that traders who use the CGP approach made less deals than traders who used the Duffie approach. In Chapter 5, the accuracy of the CGP pricing technique was evaluated in comparison to the Duffie approach and it showed that the CGP pricing technique improves considerably the prediction in comparison to the Duffie approach. Therefore, using the CGP technique, traders have a better estimate of the prices and thus they can better identify the non profitable or risky deals which results in them making less deals. Taking into account that higher numbers of CDS deals lead to a higher chance for default it is possible that the CDS contract is risky and the main reason for defaults in the simulation.

However, to be able to generalise the finding further investigation of this issue is required. As explained in Chapter 2, a CDS contract has two legs: a buyer and a seller. Observing that the CDS contract increases the risk of default, it is interesting to know which characteristic of this contract is problematic. The next section is focused on investigating the CDS contract legs in order to identify the embedded risk of CDS contracts.

7.6 Experiment 3: CAVE as an Exploration Tool

7.6.1 Experimental Setting

Experiment 2 demonstrated how the CGP price calculator improved traders' performance (decreasing default rate) by making less CDS deals. In this experiment, the aim is to study the capability of ABM models such as CAVE in explaining the reason behind the market events (e.g. default of traders) and in investigating different policies.

To do so, this section studies two scenarios: A) to identify the risky deals and, B) to study trader behaviour under a new policy. In Experiment 3A, traders are restricted to only participate in one leg of the CDS contract during the simulation. All traders use a reduced-risk strategy but each trader can

perform only one kind of CDS deal during the experiment. In other words, a trader has only the right to be a CDS buyer or a CDS seller. This experiment can help to identify which leg of the CDS contract is causing problems for traders.

In Experiment 3B, a new rule by forcing traders is applied to reserve capital for the deal that they make and only allow them to make a deal if their capital is above a certain threshold. The following three conditions are applied to the model:

- The trader capital will be virtually decreased by the compensation amount to reserve capital for the possible default event. In the case of default, the reserved capital would be paid to the corresponding CDS buyer. The normal expiration of the contract leads to freeing the reserved capital.
- The credit rating agent uses the virtual capital of traders for ranking purpose thus projecting the true financial situation of a trader.
- A trader can only sell a CDS contract if and only if his virtual capital remains above 20% of his capital amount after the sell.

All other experimental parameters are as in Experiment 1.

7.6.2 Results

Experiment 3A

Table 7.7 presents the results of our Experiment 3A. In this experiment traders can only participate in one leg of the CDS contract. This means they can choose to be a CDS buyer or CDS seller but they cannot change their role during the simulation. The aim is to investigate whether CDS buyers and sellers are exposed to a similar risks and if not then which leg of the CDS contract makes problems for traders.

We can first observe that the number of defaults is like in Experiment 2 down from the number of defaults experienced in Experiment 1 (Table 7.2). For the real market the default rate is down from 15.68 to 13.34, for the normal market it is down from 14.8 to 11.32) and for the crisis market it is down from 20.34 to 12.28. This is a hint that the imposed restriction has some mild regulatory effect on the market.

Table 7.7: Trader Default Rates.

	Real Market	Normal Market	Crisis Market
Mean	13.34	11.32	12.28
STD	1.9231	1.9213	2.4829

The dependence on the initial credit rating on the default rate was also investigated. This time, in contrast to Experiment 2, there is no striking difference to the results in Experiment 1 and the distribution is only slightly skewed towards traders with initial credit rating B. So, it can be said that the modification affects the traders independently of their initial credit rating.

It is significant if there is a difference in default rates if CDS buyers are compared with CDS sellers and in fact it was found that there is a striking difference in the number of defaults in dependence of the

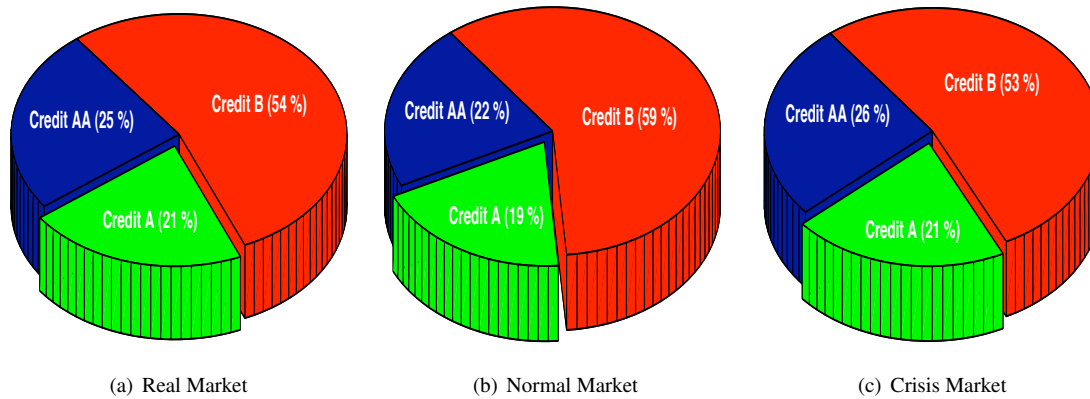


Figure 7.11: Trader Default Rates Based on the Initial Credit Rating.

role of the trader. Figure 7.12 shows these numbers and in all three markets CDS sellers are responsible for around 85% of the defaults while CDS buyers are on average only responsible around 15%. This is a strong indicator that CDS sellers carry a significant amount of risk in comparison to CDS buyers.

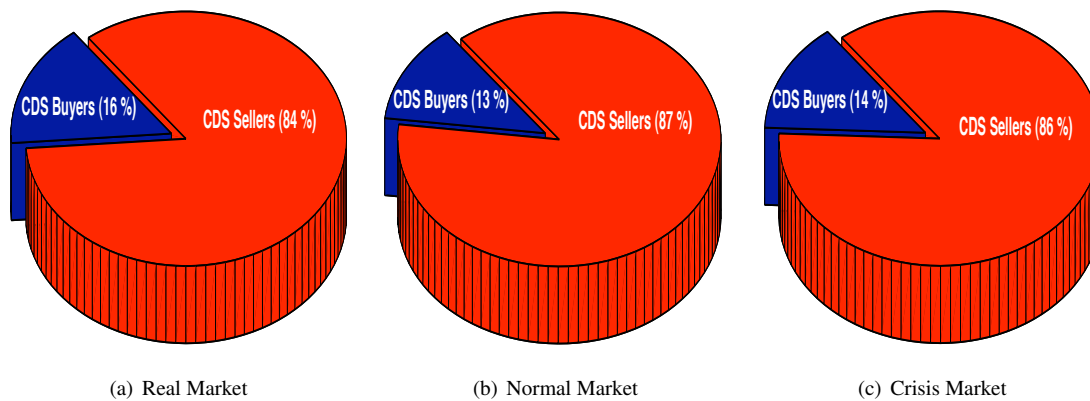


Figure 7.12: Trader Default Rates Based on Their Role: Buyer/Seller

The main question is now why the default rate of CDS sellers is so much higher than the default rate of CDS buyers when both of them are exposed to the default of the same risky asset. To answer this question 20% of the CDS sellers were reviewed and buyers activity in detail and it was found that 60% of CDS sellers need a minimum of 145% of their capital to be able to pay compensations in the case of the default which means they promised far more than they could afford. Surprisingly, these traders have a very good initial credit ranking like AA, A or B. In contrast, the CDS buyers capital and their credit ranking matches.

There is also another effect in favour of CDS buyers: In the instance of a default CDS buyers are not only not obliged to pay any money but they also receive 60% (or the agreed recovery rate) of their money back through compensation from CDS sellers.

Based on this result, selling a CDS contract has no impact on the trader financial status and its credit rating. This is a dangerous situation since a trader can sell contracts regardless of its financial situation in an attempt to make money through the CDS premiums. Moreover, this is extremely misleading for

CDS buyers who rely on the credit rating factor for choosing a CDS seller.

This anomaly can be explained through the nature of CDS contracts. As was discussed in Section 2.1, a CDS seller is not required to maintain any capital reserves to guarantee payment of claims in the case of a risky default. This means that a trader can sell a CDS contract even if he has no money to pay in the situation of a default. This is one of the CDS market problems as selling a CDS contract does not affect the trader's assets and therefore the credit rating agencies are not capable of assessing the trader's business. In this situation, the CDS buyer is not only taking the risk of a risky company default but also he is exposed to the CDS sellers risk of a default. Moreover, a CDS seller has no restriction on the number of CDS contracts he sells and this brings uncertainty to the market by providing a fake kind of protection.

A final question to be examined in this experiment is how the CDS contract selling rate varies over the three markets. Table 7.8 shows the resulting numbers. Most CDS contracts are sold in the normal market.

Table 7.8: Number of CDS Contracts Sold by Traders.

	Real Market	Normal Market	Crisis Market
Mean	166.78	302.82	123.54
STD	27.616	61.153	24.251

Experiment 3B

Having analysed what causes the traders to default in Experiment 3A, in Experiment 3B the possibility of using different policies for preventing this problem is investigated. In particular, traders are now forced to reserve capital for CDS deals as described in Section 7.6.1 which prevents traders from overselling the contracts. Furthermore, the true credit ratings of the traders are now available to the market which allows for a better estimate of the contracts.

Table 7.9 presents the default rates after applying the new conditions. There is a very strong decrease in defaults in all three markets: In the real world market we see an average decrease from 15.68 to 3.66 defaults, in the normal market simulation we have an average decrease from 14.8 to 3.64 and in the crisis market an average decrease from 20.34 to 3. All decreases are significant to an $\alpha = 5\%$ level and in fact the critical values are far lower (real data: 10^{-45} , normal market: 10^{-47} and crisis market: 10^{-74}) and the results are highly significant as one would expect. This is good news since easy policy adaptations allow for a massive gain in stability in the market.

Table 7.9: Trader Default Rates

	Real Market	Normal Market	Crisis Market
Mean	3.66	3.64	3
STD	2.0663	1.5221	1.6036

This positive effect also shows in the number of CDS sellers who default. This rate decreases to 0% with the new policy. The 0% defaulting rate is reasonable in this experiment since the traders business

deals are limited to loan deals and CDS selling contracts. Neither of these two deals can contribute to a default event since traders are forced to make these deals only if they have enough money. In addition, after making these deals traders only make money through the interest payments and the premiums. They do not have any further payment obligations in any case thus a default event cannot result in a chain of defaults of multiple traders.

Table 7.10: Number of CDS Contracts Sold by Traders.

	Real Market	Normal Market	Crisis Market
Mean	72.14	97.82	44.26
STD	18.148	24.09	12.084

The detailed behaviour of the systems was analysed. Starting with the number of CDS contracts that are sold (Table 7.10) it can be observed that this number is down by around 40% through the new policy compared to the numbers observed in Experiment 1.

Looking next at the distribution of defaults based on the initial credit rating category it can now be seen that the lion's share of defaults is with traders with initial credit ranking B. Hardly any traders with initial credit ranking A default and no traders with initial credit rating AA.

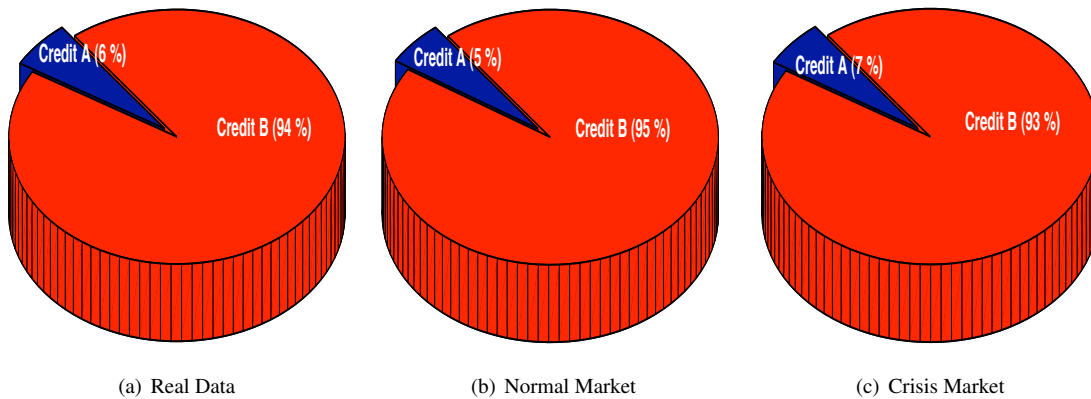


Figure 7.13: Trader Default Rates Based on Their Initial Credit Rank.

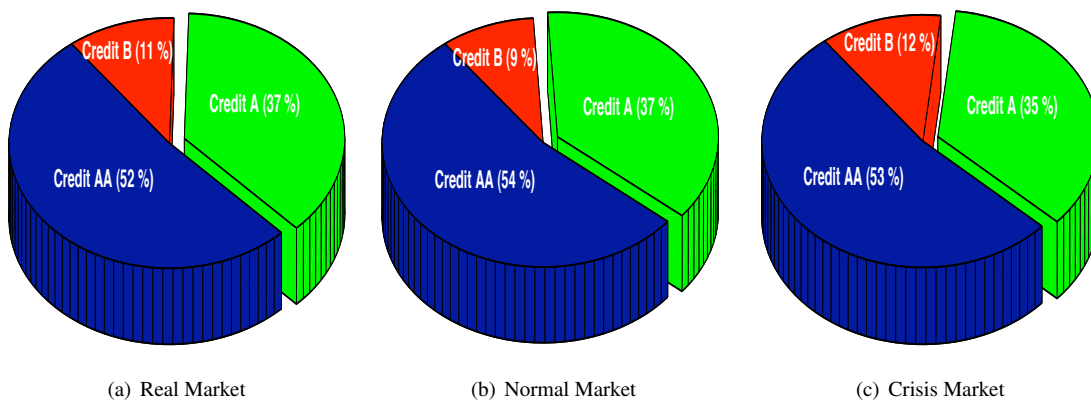


Figure 7.14: Number of CDS Contracts Sold Based on Initial Credit Ranking of Traders.

While the number of CDS contracts went down significantly under the new policy the distribution based on the initial credit ranking does not differ significantly from the one in Experiment 1 as shown in Figure 7.14. So the new policy has no effect on the distribution of defaults over credit ratings. This is not surprising since the stabilising effect should not depend on the initial credit rating. If the initial capital is too low, then no deals can be made and a trader needs to first accumulate sufficient capital which will have a similar effect to starting in a better credit rating category.

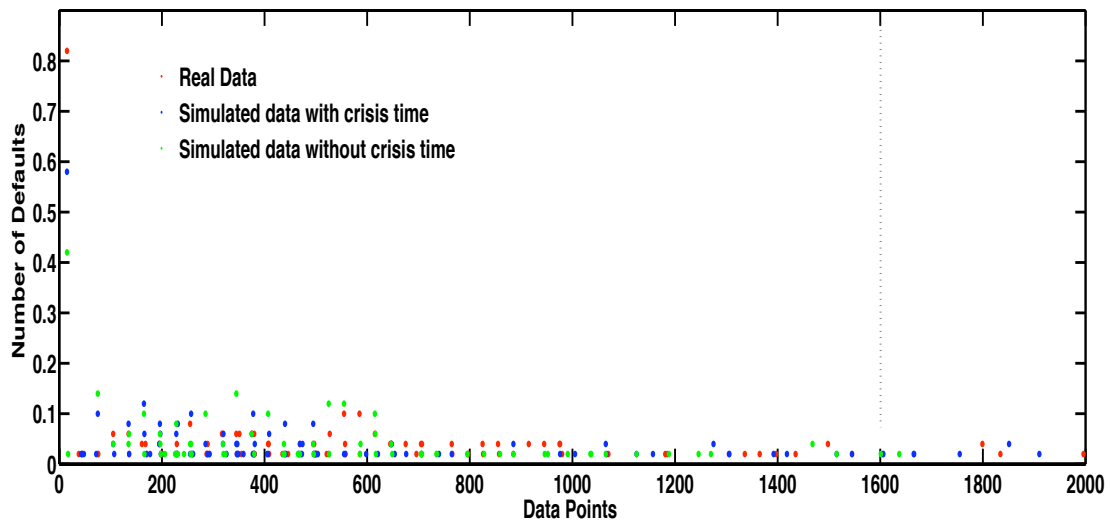


Figure 7.15: Trader Default

The final evaluation was performed for the time of defaults of the traders as shown in Figure 7.15. There is no significant behaviour in this plot. The number of defaults is consistently low, decreasing slightly towards the end of the simulation.

7.7 Evaluation and Discussion

As discussed in Section 2.1, CDS was identified as the main component of 2007-2012 crisis. It was widely agreed that the CDS market suffers from low transparency and fundamental infrastructure problems [Anon, 2008]. In 2010, Mr Trichet also highlighted the danger of relying on a single tool and pointed out the urgent need for developing complementary tools (annual European Central Bank (ECB) conference) [Trichet, 2010]. In this chapter, we presented the CAVE model as a reliable tool for investigating the CDS market under different scenarios (Section 7.4.2). We further showed how the traders performance improved (lower default rate) by using a better pricing technique in comparison to the pricing Benchmark (Section 7.5.2). Our result is confirmed by Declaration of the Summit on Financial Markets and the World Economy [Anon, 2008] where the insufficiency of CDS pricing is discussed. The writer argued that CDS prices show only small relation to the associated risk. This is not a surprise when the theoretical CDS pricing (see Chapter 5) shows no or little connection to the real CDS market price. Although those who trade CDS contracts might have sophisticated techniques for pricing them. But not sharing this information for the sake of financial benefit only decreases the market transparency.

In this chapter, we demonstrated how the CGP technique can be used together with the AB model to

reduce the randomness and form a more realistic AB model. Our approach is a response to Mr Trichets call for developing complementary tools. Section 7.6.2 presented the problematic nature of the CDS contract. This problem is discussed in an article by Matthew Philips, where he explains how the CDS becomes dangerous while it is supposed to be an insurance against risky loans [Philips, 2008]. The risk of counterparties defaulting has been intensified during the recent financial crisis, particularly because Lehman Brothers and AIG were involved as counterparties in a very large number of CDS transactions. This is an empirical example of systematic risk, that is risk which threatens an entire market, and a number of commentators have claimed that size and deregulation of the CDS market have increased this systemic risk [Nakisa, 2011] and exacerbated the 2008 global financial crisis by hastening the demise of companies such as Lehman Brothers and AIG [Philips, 2008].

We successfully illustrated how our model illustrated traders to better estimate the CDS price and counterparty risk therefore making less CDS deals and reducing the default chance. In addition, we demonstrated how AB models such as CAVE can help policy makers in testing different regulation (Section 7.6.2). This is a crucial benefit of this simulation model since a rule or a policy can be investigated thoroughly under different conditions before it is applied in the real world. This is a necessary infrastructure not only for the financial market but also for many other domains such as social health care and education systems. Such a model can improve transparency and increase regulation of the system thus avoiding unexpected events.

7.8 Conclusion

In chapter 4, 5, and 6, we presented the successful simulation of data using Levy processes, illustrating the reliability of the CGP approach for pricing CDS contracts, and demonstrating how AB models can simulate CDS markets.

In this Chapter, we illustrated how the cooperation of these three approaches can form an experimental tool for studying and investigating the CDS market. By comparing the results of different simulation periods we showed how the limitation of real-world data can restricts our views and understanding as well as demonstrating the advantage of integrating a data generative model into an AB model for studying different scenarios. The simulated data enabled us to monitor the traders performance for a long period of time thus providing a better understanding of market events as well as highlighting the irregularity of some traders performance with respect to their peers. Then we showed how our complementary pricing tool helps the better pricing of CDS contracts thus improving the traders overall performance. Finally, our last two experiments presented the capability of our CAVE model for identifying the problems and assessing different policies. Our results are validated by the means of experiments and linking our findings to real world events. The CAVE model successfully replicated some of the fundamental CDS market facts and proved its capability as an experimental tool for studying the market.

The next Chapter provides a summary and a critical evaluation of contributions. This is followed by presenting possibilities for future work and research vision.

Chapter 8

Conclusions

This chapter provides a summary (Section 8.1) and a critical evaluation of contributions (Section 8.2) that are presented in this thesis. We further discuss the future direction of this research stream, see Section 8.3.

8.1 Summary of Contributions

A critical review of previous studies shows that despite the known limitation of traditional macroeconomic models, the importance of newly established approaches is ignored in the world economy. This thesis was motivated by the need to develop new approaches to enable a better understanding of the complex world economy. In particular, this thesis focused on addressing three challenges in Credit Default Swap (CDS) research, namely the lack of sufficient data to support research, lack of efficient CDS pricing technique to be integrated into agent based model, and lack of practical CDS market experimental model which are faced by designers of CDS investigation tools.

We defined our overall research objective as studying the modelling of the CDS market using Agent Based (AB) simulation while utilising complementary tools such as a CDS price calculator and a data generative model which can offer an effective and efficient experimental tool for investigating the CDS market. Consequently, this thesis focuses on three primarily objectives: 1) Studying the challenge of simulating data with a focus on incorporating the real-world data characteristics, integrating a market common trend, overcoming the problems of limited data length, limited sampling pool, and limited scenarios. 2) Studying the challenge of the CDS pricing with a focus on providing a price calculator tool which can be integrated with a multi-agent system. 3) Studying the challenge of modelling the CDS market using AB simulation while utilising complementary tools, and with a focus on simulating market participants, events, and participants negotiation process. We achieved our objectives by means of the following contributions:

- We developed a stochastic process (Chapter 4) based on Lévy process [Applebaum, 2004] and maximum likelihood method [Applebaum, 2004] for designing a data generative model that uses the attributes of real-world data in order to generate financial data that resemble real data with the purpose of overcoming the lack of real-world data for long simulation times (20+ years). This is the first time that this technique has been used for feeding an AB model. Our model

is designed in two forms: independent and dependent. The independent model (Section 4.3.3) is capable of simulating data for factors such as interest rate. The dependant model (Section 4.3.4), takes a common market factor (market trend) as an input to simulate an industry sector behaviour. Additionally, our model is capable of generating different scenarios using different data characteristics. We classified the real data in to two parts *Normal time*” and *Crisis time*” based on the key differences in data characteristics (Section 4.2) thus enabling the simulation of different scenarios by reproducing the crisis period or normal period as many times as desired. Moreover, the randomness property of the Lévy process allows the generation of unlimited samples from one dataset. We validated this approach by means of comparative evaluation (Section 4.4). The outcome of this contribution is in the preparation process for the journal of Autonomous Agents and Multi-Agent Systems [Zangeneh and Bentley, 2014a].

- We presented a CDS price calculator (Chapter 5), using Cartesian Genetic Programming (CGP) approach [Barricelli, 1954], to be used by modelled traders in the virtual economy in which the best function for pricing each individual CDS contract is discovered based on the available financial history (e.g. bond price and interest rate) of referenced company (Section 5.1). The derived solutions, chromosomes, are stored in a file and can be accessed by standard programs. This is the first time that the CGP technique has been used for pricing CDS contracts. Moreover, we provide a study comparison of an evolutionary (CGP approach) and a well established non-evolutionary regression tool (Gaussian Process Regression (GPR) approach [Andrawis et al., 2011], see Section 5.2) for pricing CDS contracts and their compatibility with AB models. We validated this approach by means of a comparative evaluation (Section 5.3). The outcome of this contribution is published in different conference proceedings of Computational and Financial Econometrics [Zangeneh and Bentley, 2009] and Parallel Problem Solving From Nature [Zangeneh and Bentley, 2010]. The comparative study of CGP and GPR is in preparation for the journal of Evolutionary Computing [Zangeneh and Bentley, 2014b].
- We designed and presented a CDS market experimental model using the AB technique (Chapter 6). In particular, we simulated market participants such as risky companies and a regulator (Section 6.4.1), traders (Section 6.4.2), and a credit rating agent (Section 6.4.1). The market participants properties such as strategies and negotiation process is also explained in detail. Two types of market events (Section 6.4.3), structural approach and reduced-form approach, simulated traders and risky companies defaults. Our model demonstrated how an AB model can help to analyse different elements of the CDS market. This is the first time that an AB model has been utilised for studying the CDS market fundamentals, behaviour, and events. We validated this approach by means of experiments and comparative evaluation (Section 6.8). The outcome of this contribution is in the process of preparation for the journal of Autonomous Agents and Multi-Agent Systems [Zangeneh and Bentley, 2014a].
- We presented a novel demonstration of integrating statistical techniques and regression approaches

into an AB model (Chapter 7). A novel methodology is proposed in order to create a general purpose model for investigating CDS market (Section 7.1). Furthermore, we developed chromosome reader functions from the original CGP program. This function has been added to the Credit Default Swap Agent-Based model and acts as translator to calculate the price of CDS contracts using the stored chromosome (Section 7.1).

Three different sets of data (Real Market, Normal Market, and Crisis Market) were prepared for the purpose of our experiments (Section 7.3). We validated this approach by means of experiments and comparative evaluation (Section 7.7). The outcome of this contribution is in the process of preparation for the journal of Autonomous Agents and Multi-Agent Systems [Zangeneh and Bentley, 2014a].

8.2 Critical Evaluation

We evaluated the contributions of this thesis by means of simulation over a specific set of datasets (real data and simulated data) and by linking our findings to discussion in the real world or events when appropriate. This work did not consider evidence gathered through comparative studies for there may be no clear sense in which one approach is better or worse than another because different modelling approaches may solve the challenges with very different and incompatible methods. Therefore, we expect the reported validation to hold only in scenarios where the market follows the same characteristics of those used in the tested models and datasets. However, if these approaches are applied to different problems using different datasets, the experimenter can adapt the model to a new problem by adjusting the models parameters.

Lack of sufficient data to support research. The data generative model is a statistical approach which uses the maximum likelihood technique for optimizing the main variables such as sigma, see Section 4.3.3. The maximum likelihood technique is sensitive to input data and the two main parameters of σ_B and σ_H . Therefore, the experimenter should tune the model parameters to the chosen dataset.

Lack of efficient CDS pricing technique. As with most machine learning approaches, the solutions derived from the CGP approach are dependent on the models settings such as population size, mutation rate, number of generations, number of nodes, and functions set. Changing any of these parameters may lead to different solutions, thus not guaranteeing the expected precision and efficiency of the derived solution. Additionally, different datasets might demand different settings based on the data characteristics.

Lack of practical CDS market experimental model. The same principle is true for the CDS experimental model and the Credit Default Swap Agent-Based Virtual Economy (CAVE) model where changing the models fundamental settings could lead to different result and insights. The various number of techniques (e.g. pricing techniques or traders risk preferences) and settings (e.g. number of risky companies or number of traders in the society) are highly beneficial since it enables

market investigation under different conditions. For such models to be used in the real world, it would also be important to calibrate all settings and outputs with as much real world data as possible, iteratively revising settings until suitable correlation is achieved.

8.3 Future Work

This thesis offers various contributions with regard to the Agent-Based Computational Economics (ACE) field. This field could be further advanced in a number of ways to improve the performance of the techniques and capabilities of approaches proposed in this thesis.

Lack of sufficient data to support research. The data generative model described in Chapter 4 has a minor shortcoming, in that the rate λ for the jumps stays close to the boundary of two which were defined. This is most likely not a shortcoming of the Maximum likelihood approach but of the concrete optimiser that we used: the Matlab optimization toolbox. The toolbox is a general optimisation toolbox that is not adapted to our concrete problem at hand. It is very likely that this optimisation routine finds only local optima and not a global optima. The problem could be addressed by using optimisation approaches that are better suited to this model. A particularly promising algorithm to address this problem is called the Expectation-Maximization algorithm (EM, [Dempster et al., 1977]). This algorithm is well-suited for time series models. In fact, the EM is the standard algorithm with which HMMs are trained. In the HMM literature the EM algorithm is also known as the Baum-Welch algorithm [Baum et al., 1970]. The close link between our model and HMMs suggests that the EM is a good candidate for future research to improve on the shortcoming in the rate parameter λ .

Lack of efficient CDS pricing technique. A very natural extension to the work described in Chapter 5 would be designing a model to analyse the CGP chromosomes. In this work we designed a chromosome translator for translating the chromosomes into mathematical notation. However, this chromosome translator model is incapable of analysing the solution in details for providing insights regarding the pricing paradigm. Additionally, it would be interesting to integrate the whole pricing calculator model into the AB model for enabling the online pricing of CDS. The main issue for this integration can be defined as computational time since the CGP is a very slow technique. Based on CAVE model parameters such as the number of traders and the number of risky companies and, given the slow speed of the CGP approach, the simulation might simply take years before providing any result, especially if the artificial traders need to test different CGP model parameters to find the best setting. To make this method viable, a new parallel architecture would be needed to speed up execution times.

Lack of practical CDS market experimental model. In this thesis, we have introduced the first steps in tackling the problem of designing a CDS experimental tool to overcome some of the traditional macroeconomic models limitations and help improve investigation thus leading to a better understanding of complex financial markets. In doing so, we incorporated some real data as well as

simulating the data using a well-known Levy process. We also utilised a CGP technique to provide a more accurate pricing tool for the modelled traders. However, the model is not yet the exact replication of the real CDS market due to the level of abstraction applied as well as it involving random elements. Moreover, a more comprehensive real financial database is needed to enable the more realistic replication of CAVE objects such as traders, and risky companies. Including all the details increases the complexity of the model, but not including them does not allow the true simulation of the market and its characteristics. Therefore, the results might not match the real world events in detail. It remains open for future work, to improve the model by reducing the randomness, abstraction and obtaining more real data. Additionally, our CAVE model produces results in large volume, however if the experimenter analyses these results it is time consuming and might introduce human error. Designing an automated data analysis system which is customised for the CAVE output format will highly increase the output analysis speed thus allowing the experimenter to run more experiments under different model settings, thus providing more reliable insights in a shorter time.

8.4 Summary and Conclusions

The aim of this work as stated in section 1.2 was:

To investigate the viability of approaches that address the three challenges of lack of sufficient data to support research, lack of efficient CDS pricing technique to be integrated into agent based model, and lack of practical CDS market experimental model, that are faced by designers of CDS investigation tools.”

This thesis has clearly achieved its aim by demonstrating three viable approaches that address the three challenges.

A data generative engine is presented, using the well-known Lévy process and maximum likelihood method, for producing realistic artificial data which enables open-ended simulation as well as designing different scenarios. A novel CDS price calculator is proposed, using the CGP approach to enable a customised calculation of CDS contracts. An AB model is designed for simulating the CDS market thus enabling the investigation of the CDS market under different scenarios and laboratory conditions. Finally a novel framework was presented for integrating these three solutions.

No single PhD can solve the problem of modelling the CDS market, but this work has made some significant steps towards the development of successful models in the future.

Acronyms

AB Agent Based.

ABM Agent-Based Modeling.

ACE Agent-Based Computational Economics.

ANN Artificial Neural Network.

AR Autoregressive.

bp Basis Point.

CAVE Credit Default Swap Agent-Based Virtual Economy.

CD Credit Derivative.

CDS Credit Default Swap.

CGP Cartesian Genetic Programming.

CRISIS Complexity Research Initiative for Systemic Instabilities.

DSGE Dynamic Stochastic General Equilibrium.

EA Evolutionary Algorithms.

EC Evolutionary Computation.

ECB European Central Bank.

FCFS First-come, first-served.

GDT Genetic Decision Tree.

GP Genetic Programming.

GPR Gaussian Process Regression.

HMM Hidden-Markov Model.

K.I.S.S. Keep It Simple Stupid.

LLN Law of Large Numbers.

OTC Over-the-counter.

PBIL Population Based Incremental Learning.

SMA Simple Moving Average.

SOM Self-Organizing Maps.

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